

PRODOTTI NOTEVOLI

ESPRESSIONI

1. $(a - b)^2 - (a - 2b)^2 + (a - b)(a + 2b) =$
 $= a^2 - 2ab + b^2 - a^2 + 4ab - 4b^2 + a^2 + 2ab - ab - 2b^2 = a^2 + 3ab - 5b^2$
2. $(x - 2y)^2 + (x - 3y)(x + 3y) - (x + 2y)(x - 3y) =$
 $= x^2 - 4xy + 4y^2 + x^2 - 9y^2 - x^2 + 3xy - 2xy + 6y^2 = x^2 - 3xy + y^2$
3. $(a - 1)^2 + (2a - 1)(2a + 1) - (-2a + 1)(-2a - 1) + 2a =$
 $= a^2 - 2a + 1 + 4a^2 - 1 - 4a^2 + 1 + 2a = a^2 + 1$
4. $\left(2a - \frac{1}{2}\right)\left(2a + \frac{1}{2}\right) + \left(1 - \frac{1}{2}a\right)^2 - \left(\frac{a}{2} - 2\right)\left(\frac{a}{2} + 2\right) + a =$
 $= 4a^2 - \frac{1}{4} + 1 - a + \frac{1}{4}a^2 - \frac{1}{4}a^2 + 4 + a = 4a^2 + \frac{19}{4}$
5. $(2a - x)(2a + x) + (a - 2x)^2 - (x - 2a)^2 =$
 $= 4a^2 - x^2 + a^2 - 4ax + 4x^2 - x^2 + 4ax - 4a^2 = a^2 + 2x^2$
6. $(2a^2 - 1)(a + 1) + (3a + 2)(2 - 3a) - 2a^3 + 7a(a - 1) =$
 $= 2a^3 + 2a^2 - a - 1 + 4 - 9a^2 - 2a^3 + 7a^2 - 7a = -8a + 3$
7. $4(a + b)^2 + (-2a + b)^2 - b^2 - (a + 2b)^2 =$
 $= 4a^2 + 8ab + 4b^2 + 4a^2 - 4ab + b^2 - b^2 - a^2 - 4ab - 4b^2 = 7a^2$
8. $(2 + a - b)(2 - a + b) + (a - b)^2 = 4 - (a - b)^2 + (a - b)^2 = 4$
9. $(x + y + z)^2 + (x - y)^2 + (y - z)^2 + (z - x)^2 =$
 $= x^2 + y^2 + z^2 + 2xy + 2xz + 2yz + x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2xz + x^2 = 3x^2 + 3y^2 + 3z^2$
10. $(a - 2b)^2(a + 2b)^2 - (a^2 - ab - b^2)(a^2 - ab + b^2) + (-3ab)^2 =$
 $= (a^2 - 4b^2)^2 - [(a^2 - ab)^2 - b^4] + 9a^2b^2 =$
 $= a^4 - 8a^2b^2 + 16b^4 - a^4 + 2a^3b - a^2b^2 + b^4 + 9a^2b^2 = 17b^4 + 2a^3b$
11. $(x + 2y)^3 - (x - 2y)^3 - 3y(-2x)^2 =$
 $= x^3 + 6x^2y + 12xy^2 + 8y^3 - x^3 + 6x^2y - 12xy^2 + 8y^3 - 12x^2y = 16y^3$
12. $(a^2 - 1)(a^2 + 1)(a^2 + 2) - (a^2 - 1)^3 - (2a^2 - 1)^2 =$
 $= (a^4 - 1)(a^2 + 2) - (a^6 - 3a^4 + 3a^2 - 1) - (4a^4 - 4a^2 + 1) =$
 $= a^6 + 2a^4 - a^2 - 2 - a^6 + 3a^4 - 3a^2 + 1 - 4a^4 + 4a^2 - 1 = a^4 - 2$
13. $(a + b + c)(a + b - c) - (a - b + c)(b + c - a) + (a + b + c)^2 =$
 $= (a + b)^2 - c^2 - [c^2 - (a - b)^2] + a^2 + b^2 + c^2 + 2ab + 2ac + 2bc =$
 $= a^2 + b^2 + 2ab - c^2 - c^2 + a^2 - 2ab + b^2 + a^2 + b^2 + c^2 + 2ab + 2ac + 2bc =$
 $= 3a^2 + 3b^2 - c^2 + 2ab + 2ac + 2bc$

14. $(-x + y + z + t)(x - y + z + t) + (x + y - z + t)(x + y + z - t) =$
 $= (z + t)^2 - (x - y)^2 + (x + y)^2 - (z - t)^2 =$
 $= z^2 + 2zt + t^2 - x^2 + 2xy - y^2 + x^2 + 2xy + y^2 - z^2 + 2zt - t^2 = 4xy + 4zt$
15. $(a - b)^3 + 3(a - b)^2(a + b) + 3(a - b)(a + b)^2 + (a + b)^3 =$
 $= [(a - b) + (a + b)]^3 = (a - b + a + b)^3 = (2a)^3 = 8a^3$
16. $3(x - y)^2(x + y) + 3(x + y)^2(y - x) - 6y(x + y)(-x + y) =$
 $= 3(x - y)(x - y)(x + y) + 3(x + y)(x + y)(y - x) - 6y(-x^2 + y^2) =$
 $= 3(x - y)(x^2 - y^2) + 3(x + y)(y^2 - x^2) + 6x^2y - 6y^3 =$
 $= 3x^3 - 3xy^2 - 3x^2y + 3y^3 + 3xy^2 - 3x^3 + 3y^3 - 3x^2y + 6x^2y - 6y^3 = 0$
17. $[x^2 + 2(x - 1)]^2 - 4x^2(x - 1) - [2(x - 1)]^2 - (x^2 - 1)^2 =$
 $= (x^2 + 2x - 2)^2 - 4x^3 + 4x^2 - (2x - 2)^2 - x^4 + 2x^2 - 1 =$
 $= x^4 + (2x - 2)^2 + 2x^2(2x - 2) - 4x^3 + 4x^2 - (2x - 2)^2 - x^4 + 2x^2 - 1 =$
 $= x^4 + 4x^3 - 4x^2 - 4x^3 + 4x^2 - x^4 + 2x^2 - 1 = 2x^2 - 1$
18. $(a^2 - 2a)^3 + a(2a^2 + 3a)^2 - 2a^3\left(2a - \frac{1}{2}\right)^2 - a^4(a + 2)(a - 12) =$
 $= a^6 - 6a^5 + 12a^4 - 8a^3 + a(4a^4 + 12a^3 + 9a^2) - 2a^3\left(4a^2 - 2a + \frac{1}{4}\right) - a^4(a^2 - 10a - 24) =$
 $= a^6 - 6a^5 + 12a^4 - 8a^3 + 4a^5 + 12a^4 + 9a^3 - 8a^5 + 4a^4 - \frac{1}{2}a^3 - a^6 + 10a^5 + 24a^4 = +52a^4 + \frac{1}{2}a^3$
19. $\left\{ \left[2a^2 - \left(a - \frac{1}{2}b \right) \left(a + \frac{1}{2}b \right) \right]^2 - \frac{1}{16}b^4 \right\}^2 - \left[\frac{1}{2}a^2(2a^2 + b^2) \right]^2 =$
 $= \left\{ \left[2a^2 - \left(a^2 - \frac{1}{4}b^2 \right) \right]^2 - \frac{1}{16}b^4 \right\}^2 - \left[a^4 + \frac{1}{2}a^2b^2 \right]^2 =$
 $= \left\{ \left[2a^2 - a^2 + \frac{1}{4}b^2 \right]^2 - \frac{1}{16}b^4 \right\}^2 - \left[a^4 + \frac{1}{2}a^2b^2 \right]^2 =$
 $= \left\{ a^4 + \frac{1}{2}a^2b^2 + \frac{1}{16}b^4 - \frac{1}{16}b^4 \right\}^2 - \left[a^4 + \frac{1}{2}a^2b^2 \right]^2 =$
 $= \left\{ a^4 + \frac{1}{2}a^2b^2 \right\}^2 - \left[a^4 + \frac{1}{2}a^2b^2 \right]^2 = 0$
20. $\left(y - \frac{1}{2}x \right)^3 - \left(-\frac{1}{2}x - y \right)^3 - 6 \left[y \left(\frac{1}{2}x - y \right)^2 + x(-y)^2 \right] =$
 $= y^3 - \frac{3}{2}xy^2 + \frac{3}{4}x^2y - \frac{1}{8}x^3 + \frac{1}{8}x^3 + \frac{3}{4}x^2y + \frac{3}{2}xy^2 + y^3 - 6 \left[y \left(\frac{1}{4}x^2 - xy + y^2 \right) + xy^2 \right] =$
 $= 2y^3 + \frac{3}{2}x^2y - 6 \left[\frac{1}{4}x^2y - xy^2 + y^3 + xy^2 \right] = 2y^3 + \frac{3}{2}x^2y - \frac{3}{2}x^2y - 6y^3 = -4y^3$

21. $5b \left(1 + \frac{1}{4}b \right) + (b^2 + b - 1)^2 - (b + 1)^3 - b^2 \left(b + \frac{1}{2} \right) =$
 $= 5b + \frac{5}{4}b^2 + b^4 + b^2 + 1 + 2b^3 - 2b^2 - 2b - b^3 - 3b^2 - 3b - 1 - b^3 - \frac{1}{2}b^2 = -\frac{13}{4}b^2 + b^4$
22. $3a^2b^2 - (a^2 - 3b^2)^2 - b^2 \left(2a - \frac{1}{4}b \right)^2 + \left[\left(a - \frac{1}{2}b \right) \left(a + \frac{1}{2}b \right) \right]^2 =$
 $= 3a^2b^2 - a^4 - 9b^4 + 6a^2b^2 - b^2 \left(4a^2 - ab + \frac{1}{16}b^2 \right) + \left[a^2 - \frac{1}{4}b^2 \right]^2 =$
 $= 9a^2b^2 - a^4 - 9b^4 - 4a^2b^2 + ab^3 - \frac{1}{16}b^4 + a^4 - \frac{1}{2}a^2b^2 + \frac{1}{16}b^4 = -9b^4 + ab^3 + \frac{9}{2}a^2b^2$
23. $\{[(x + 3a)(x + 2a) - (x - 3a)^2] : (-3a)\}^2 - a(a - 7x) =$
 $= \{[x^2 + 5ax + 6a^2 - x^2 - 9a^2 + 6ax] : (-3a)\}^2 - a^2 + 7ax =$
 $= \{[-3a^2 + 11ax] : (-3a)\}^2 - a^2 + 7ax =$
 $= \left(a - \frac{11}{3}x \right)^2 - a^2 + 7ax = a^2 - \frac{22}{3}ax + \frac{121}{9}x^2 - a^2 + 7ax = \frac{121}{9}x^2 - \frac{1}{3}ax$
24. $[a^2(a - 1) - a(a^2 + 1)]^2 - a^2(a + 1)^2 + \left(-\frac{1}{2}a \right)^3 =$
 $= [a^3 - a^2 - a^3 - a]^2 - a^2(a^2 + 2a + 1)^2 + \left(-\frac{1}{8}a^3 \right) =$
 $= a^4 + 2a^3 + a^2 - a^4 - 2a^3 - a^2 - \frac{1}{8}a^3 = -\frac{1}{8}a^3$