FRAZIONI ALGEBRICHE

1. $\frac{x+2a}{a^2-x^2}-\frac{x-2a}{x^2+3ax-4a^2}+\frac{5a^2+5ax+3x^2}{x^3+4ax^2-a^2x-4a^3}$

$$=-\frac{x+2a}{(x-a)(x+a)}-\frac{x-2a}{(x+4a)(x-a)}+\frac{5a^2+5ax+3x^2}{x^2(x+4a)-a^2(x+4a)}=$$

$$=-\frac{x+2a}{(x-a)(x+a)}-\frac{x-2a}{(x+4a)(x-a)}+\frac{5a^2+5ax+3x^2}{(x+4a)(x-a)(x+a)}= \qquad C.E.: x\neq \pm a;\ \ x\neq -4a$$

$$=\frac{-(x+2a)(x+4a)-(x-2a)(x+a)+5a^2+5ax+3x^2}{(x-a)(x+a)(x+4a)}=\frac{-(x^2+6ax+8a^2)-(x^2-ax-2a^2)+5a^2+5ax+3x^2}{(x-a)(x+a)(x+4a)}=$$

$$=\frac{-x^2-6ax-8a^2-x^2+ax+2a^2+5a^2+5ax+3x^2}{(x-a)(x+a)(x+4a)}=\frac{x^2-a^2}{(x-a)(x+a)(x+4a)}=\frac{(x-a)(x+a)}{(x-a)(x+a)(x+4a)}=\boxed{\frac{1}{x+4a}}$$

2. $\frac{2y+1}{27y^3-1}+\frac{1}{9y^2+3y+1}-\frac{2}{6y^2-5y+1}$

$$=\frac{2y+1}{(3y-1)(9y^2+3y+1)}+\frac{1}{9y^2+3y+1}-\frac{2}{6y^2-2y-3y+1}=$$

$$=\frac{2y+1}{(3y-1)(9y^2+3y+1)}+\frac{1}{9y^2+3y+1}-\frac{2}{(3y-1)(2y-1)}= \qquad C.E.: y\neq \frac{1}{3};\ \ y\neq \frac{1}{2}$$

$$=\frac{(2y+1)(2y-1)+(3y-1)(2y-1)-2\,(9y^2+3y+1)}{(3y-1)(2y-1)(9y^2+3y+1)}=$$

$$=\frac{4y^2-1+6y^2-5y+1-18y^2-6y-2}{(3y-1)(2y-1)(9y^2+3y+1)}=\frac{-8y^2-11y-2}{(3y-1)(2y-1)(9y^2+3y+1)}$$

3. $\left(\frac{a-4}{a^2-5a+6}-\frac{a+2}{a^2+a-12}\right):\frac{12}{a^2+2a-8}$

$$=\left(\frac{a-4}{(a-2)(a-3)}-\frac{a+2}{(a-3)(a+4)}\right):\frac{12}{(a+4)(a-2)}= \qquad C.E.: a\neq 2,3,-4$$

$$=\frac{(a-4)(a+4)-(a+2)(a-2)}{(a-2)(a-3)(a+4)}\cdot\frac{(a+4)(a-2)}{12}=\frac{a^2-16-a^2+4}{12\,(a-3)}=-\frac{12}{12(a-3)}=-\frac{1}{a-3}=\boxed{\frac{1}{3-a}}$$

4. $\left[\left(\frac{a}{2}-\frac{2a}{b^2}\right)^{-2}:\left(\frac{a}{2}-\frac{2a}{b^2}\right)^{-1}\right]:\left(\frac{b}{a}\cdot\frac{1}{b^2-4}\right)$

$=\left(\frac{a}{2}-\frac{2a}{b^2}\right)^{-1}:\frac{b}{a(b-2)(b+2)}=$ $\quad C.E.:\ a\neq 0; b\neq 0; b\neq \pm 2$

$=\left(\frac{ab^2-4a}{2b^2}\right)^{-1}\cdot\frac{a\,(b-2)(b+2)}{b}=\frac{2b^2}{a(b^2-4)}\cdot\frac{a(b-2)(b+2)}{b}=\mathbf{2b}$

EQUAZIONI LINEARI

5. $\frac{1-2x}{2}-\frac{(1-4x)(1-2x)}{6}=\frac{5}{6}-\frac{(2x-1)^2}{3}$

$3\,(1-2x)-(1-2x-4x+8x^2)=5-2\,(4x^2-4x+1)$

$3-6x-1+2x+4x-8x^2=5-8x^2+8x-2 \qquad -8x=1 \qquad \mathbf{x=-\frac{1}{8}}$

6. $\frac{x}{6}+\left[\left(3x-\frac{1}{3}\right)^2-\left(3x-\frac{1}{3}\right)\left(3x+\frac{1}{3}\right)\right]+\frac{7}{3}x=\frac{3x-1}{6}-\frac{7}{18}$

$3x+18\left[9x^2-2x+\frac{1}{9}-9x^2+\frac{1}{9}\right]+42\,x=3\,(3x-1)-7$

$3x+18\left[-2x+\frac{2}{9}\right]+42\,x=9x-3-7$

$3x-36\,x+4+42\,x-9x=-3-7 \qquad 0x=-14 \qquad \mathbf{\nexists x\in\mathbb{R}}$

7. $\frac{x-1}{x^2+3x}+\frac{2}{x}+\frac{9}{2x+6}=0$

$\frac{x-1}{x\,(x+3)}+\frac{2}{x}+\frac{9}{2\,(x+3)}=0 \qquad C.A.: x\neq 0; x\neq -3$

$2x-2+4x+12+9x=0 \qquad 15\,x=-10 \qquad \mathbf{x=-\frac{2}{3}}\ acc.$

8. $\left(\frac{x^3-x^2}{1-x^2}+x-1\right):\left(1-\frac{x}{x+1}\right)=x^2(-x)^{-1}-2$

$\left(\frac{x^2(x-1)}{-(x-1)(x+1)}+x-1\right):\left(\frac{x+1-x}{x+1}\right)=\frac{x^2}{-x}-2$ $\qquad C.A.: x\neq\pm1; x\neq0$

$\left(\frac{-x^2}{x+1}+x-1\right):\frac{1}{x+1}=-x-2$ $\qquad \frac{-x^2+x^2-1}{x+1}\cdot(x+1)=-x-2$

$-1=-x-2$ $\qquad x=-1 \;\; non\ acc.per\ C.A.$ $\qquad \nexists x\in\mathbb{R}$

9. Determina due numeri, sapendo che il secondo supera di 17 il triplo del primo e che la loro somma è 101.

Indichiamo i due numeri come n_1 e n_2. Se poniamo $n_1=x$, allora $n_2=17+3x$ e, dato che la loro somma è 101:

$$x+17+3x=101 \qquad 4x=84 \qquad x=21$$

I due numeri sono **21** e **80**.

10. Un numero intero è formato da due cifre la cui somma è 7. Se si sottrae al triplo della cifra delle decine la metà della cifra delle unità, si ottiene 14. Qual è il numero?

Indichiamo il numero intero in forma polinomiale: $N=10x+y$. Siccome la somma delle due cifre è 7, si ottiene che: $y=7-x$. Perciò:

$$3x-\frac{1}{2}y=14 \qquad 3x-\frac{1}{2}(7-x)=14 \qquad 6x-7+x=28 \qquad 7x=35 \qquad x=5$$

Il numero richiesto è: **52**.

DISEQUAZIONI LINEARI

11. $\left(\frac{1}{2}-x\right)^2-(x+1)^2<-\left[1-\left(\frac{2x+1}{6}\right)\right]+\frac{1+2x}{3}$

$\frac{1}{4}-x+x^2-(x^2+2x+1)<-\frac{6-2x-1}{6}+\frac{1+2x}{3}$

$3-12x+12x^2-12x^2-24x-12<-10+4x+4+8x$ $\qquad -48x<3$ $\qquad \boldsymbol{x>-\frac{1}{16}}$

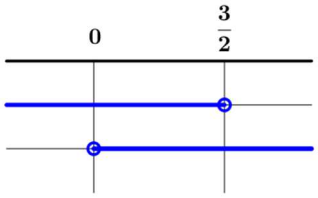
12. $\frac{x}{3}-\frac{1}{2}\left(x+\frac{2}{3}\right)<\frac{1}{3}-2\left(x+\frac{1}{3}\right)$

$\frac{x}{3}-\frac{1}{2}x-\frac{1}{3}<\frac{1}{3}-2x-\frac{2}{3}$ $\quad 2x-3x-2<2-12x-4$ $\quad 9x<0$ $\quad \boldsymbol{x<0}$

13. $\begin{cases} x^2+6x-3<2x\,(x+2)-x^2 \\ (x-2)^2+3x-3>-2x+1+x^2 \end{cases}$

$A: x^2+6x-3<2x^2+4x-x^2$ $\quad 2x<3$ $\quad x<\frac{3}{2}$

$B: x^2-4x+4+3x-3>-2x+1+x^2$ $\quad x>0$ $\quad \begin{cases} x<\frac{3}{2} \\ x>0 \end{cases}$

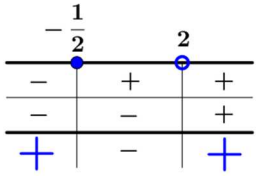


$\boldsymbol{0<x<\frac{3}{2}}$

14. $\frac{x-1}{x-2}\geq\frac{3}{4-2x}$

$\frac{x-1}{x-2}-\frac{3}{-2(-2+x)}\geq 0$ $\quad \frac{x-1}{x-2}+\frac{3}{2(x-2)}\geq 0$ $\quad \frac{2x-2+3}{2(x-2)}\geq 0$ $\quad \frac{2x+1}{x-2}\geq 0$

$N\geq 0$: $\quad x\geq -\frac{1}{2}$ $\qquad D>0$: $\quad x>2$

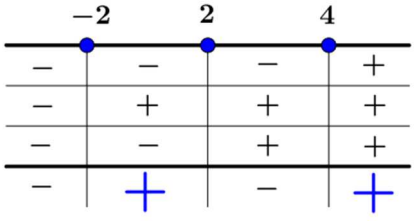


$\boldsymbol{x\leq -\frac{1}{2}}$ $\quad \vee \quad$ $\boldsymbol{x>2}$

15. $x^3-4x^2\geq 4x-16$

$x^2\,(x-4)-4\,(x-4)\geq 0$ $\quad (x-4)(x^2-4)\geq 0$ $\quad (x-4)(x+2)(x-2)\geq 0$

IF: $x\geq 4$ $\qquad IIF$: $x\geq -2$ $\qquad IIIF$: $x\geq 2$



$\boldsymbol{-2\leq x\leq 2}$ $\quad \vee \quad$ $\boldsymbol{x\geq 4}$

VALORI ASSOLUTI

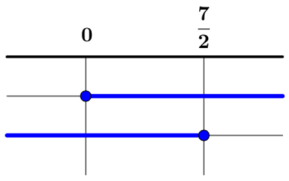
16. $|x-1| = 3x+1$

$\begin{cases} x \geq 1 \\ x-1 = 3x+1 \end{cases}$ $\begin{cases} x \geq 1 \\ x = -1 \end{cases}$ $\nexists x \in \mathbb{R}$

$\begin{cases} x < 1 \\ -x+1 = 3x+1 \end{cases}$ $\begin{cases} x < 1 \\ x = 0 \end{cases}$ $\mathbf{x = 0}$

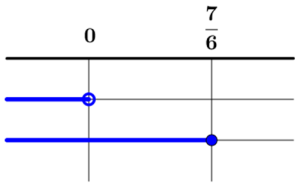
17. $\frac{1}{2} + |x| \geq 2x - 3$

$\begin{cases} x \geq 0 \\ \frac{1}{2} + x \geq 2x - 3 \end{cases}$ $\begin{cases} x \geq 0 \\ x \leq \frac{7}{2} \end{cases}$



$0 \leq x \leq \frac{7}{2}$

$\begin{cases} x < 0 \\ \frac{1}{2} - x \geq 2x - 3 \end{cases}$ $\begin{cases} x < 0 \\ x \leq \frac{7}{6} \end{cases}$



$x < 0$

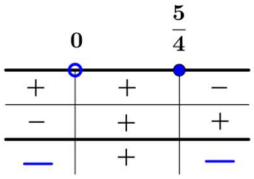
$x < 0 \quad \vee \quad 0 \leq x \leq \frac{7}{2} \quad \Rightarrow \quad \mathbf{x \leq \frac{7}{2}}$

18. $\left|\frac{5-x}{x}\right| \leq 3$

$\begin{cases} \frac{5-x}{x} \leq 3 \\ \frac{5-x}{x} \geq -3 \end{cases}$ $\begin{cases} \frac{5-x-3x}{x} \leq 0 \\ \frac{5-x+3x}{x} \geq 0 \end{cases}$ $\begin{cases} \frac{5-4x}{x} \leq 0 \\ \frac{5+2x}{x} \geq 0 \end{cases}$

$\frac{5-4x}{x} \leq 0$

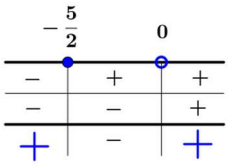
$N \geq 0: \quad x \leq \frac{5}{4} \qquad D > 0: \quad x > 0$



$x < 0 \quad \vee \quad x \geq \frac{5}{4}$

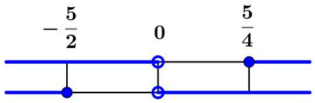
$\frac{5+2x}{x} \geq 0$

$N \geq 0: \quad x \geq -\frac{5}{2} \qquad D > 0: \quad x > 0$



$x \leq -\frac{5}{2} \quad \vee \quad x > 0$

$\begin{cases} x < 0 \quad \vee \quad x \geq \frac{5}{4} \\ x \leq -\frac{5}{2} \quad \vee \quad x > 0 \end{cases}$



$\mathbf{x \leq -\frac{5}{2} \quad \vee \quad x \geq \frac{5}{4}}$