

$$34. \quad \ln \left(x - \frac{3}{2} \right) < -\ln x$$

$$\ln \left(x - \frac{3}{2} \right) + \ln x < 0 \Rightarrow \ln \left(x - \frac{3}{2} \right) x < \ln 1$$

$$\begin{cases} x - \frac{3}{2} > 0 \\ x > 0 \\ x^2 - \frac{3}{2}x < 1 \end{cases} \Rightarrow \begin{cases} x > \frac{3}{2} \\ x > 0 \\ -\frac{1}{2} < x < 2 \end{cases} \Rightarrow \frac{3}{2} < x < 2$$

$$35. \quad \log_2(x+1) - \log_2(x-1) < 1$$

$$\log_2(x+1) < \log_2(x-1) + \log_2 2 \Rightarrow \log_2(x+1) < \log_2 2(x-1)$$

$$\begin{cases} x+1 > 0 \\ x-1 > 0 \\ x+1 < 2x-2 \end{cases} \Rightarrow \begin{cases} x > -1 \\ x > 1 \\ x > 3 \end{cases} \Rightarrow x > 3$$

$$36. \quad \frac{\log_2(3-x)}{4 + \log_{\frac{1}{2}} x} \geq 0$$

$$c.a.: \begin{cases} 3-x > 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} x < 3 \\ x > 0 \end{cases} \Rightarrow 0 < x < 3$$

$$N \geq 0: \log_2(3-x) \geq 0 \Rightarrow 3-x \geq 2^0 \Rightarrow x \leq 2$$

$$D > 0: 4 + \log_{\frac{1}{2}} x > 0 \Rightarrow \log_{\frac{1}{2}} x > -4 \Rightarrow x < 16$$

$$x \leq 2 \vee x > 16$$

$$\begin{cases} 0 < x < 3 \\ x \leq 2 \vee x > 16 \end{cases} \Rightarrow 0 < x \leq 2$$

$$37. \quad \ln^2 x - 3 \ln x - 4 < 0$$

$$c.a.: x > 0 \quad \text{Pongo: } \ln x = t \Rightarrow t^2 - 3t - 4 < 0 \Rightarrow -1 < t < 4$$

$$-1 < \ln x < 4 \Rightarrow e^{-1} < x < e^4 \Rightarrow \frac{1}{e} < x < e^4$$

$$\begin{cases} x > 0 \\ \frac{1}{e} < x < e^4 \end{cases} \Rightarrow \frac{1}{e} < x < e^4$$

38. $\log_{\frac{1}{2}}^4 x - \log_{\frac{1}{2}}^3 x \geq 0$

c.a.: $x > 0$ Pongo: $\log_{\frac{1}{2}} x = t \Rightarrow$

$t^4 - t^3 \geq 0 \Rightarrow t^3(t-1) \geq 0 \Rightarrow t \leq 0 \vee t \geq 1$

$\log_{\frac{1}{2}} x \leq 0 \vee \log_{\frac{1}{2}} x \geq 1 \Rightarrow x \leq \frac{1}{2} \vee x \geq 1$

$\begin{cases} x > 0 \\ x \leq \frac{1}{2} \vee x \geq 1 \end{cases} \Rightarrow 0 < x \leq \frac{1}{2} \vee x \geq 1$

39. $2 \log_2 x - \log_2(x+2) \leq 0$

$2 \log_2 x \leq \log_2(x+2) \Rightarrow \log_2 x^2 \leq \log_2(x+2)$

$\begin{cases} x > 0 \\ x+2 > 0 \\ x^2 \leq x+2 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > -2 \\ -1 \leq x \leq 2 \end{cases} \Rightarrow 0 < x \leq 2$

40. $\log_2 \frac{x}{4+x} - \log_2 \frac{2+x}{1-x} < -4$

$\log_2 \frac{x}{4+x} + 4 < \log_2 \frac{2+x}{1-x} \Rightarrow \log_2 \frac{x}{4+x} + \log_2 16 < \log_2 \frac{2+x}{1-x}$

$\begin{cases} \frac{x}{x+4} > 0 \\ \frac{2+x}{1-x} > 0 \\ \frac{16x}{4+x} < \frac{2+x}{1-x} \end{cases} \Rightarrow \begin{cases} x < -4 \vee x > 0 \\ -2 < x < 1 \\ \frac{17x^2 - 10x + 8}{(4+x)(1-x)} > 0 \end{cases} \Rightarrow \begin{cases} x < -4 \vee x > 0 \\ -2 < x < 1 \\ -4 < x < 1 \end{cases} \Rightarrow 0 < x < 1$

41. $\log_{\frac{1}{3}}(x+1) - 2 \log_{\frac{1}{3}}(2-x) \leq 1$

$\log_{\frac{1}{3}}(x+1) \leq 2 \log_{\frac{1}{3}}(2-x) + \log_{\frac{1}{3}} \frac{1}{3} \Rightarrow \log_{\frac{1}{3}}(x+1) \leq \log_{\frac{1}{3}} \frac{1}{3} (2-x)^2$

$\begin{cases} x+1 > 0 \\ 2-x > 0 \\ x+1 \geq \frac{1}{3} (2-x)^2 \end{cases} \Rightarrow \begin{cases} x > -1 \\ x < 2 \\ x^2 - 7x + 1 \leq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x < 2 \\ \frac{7-3\sqrt{5}}{2} \leq x \leq \frac{7+3\sqrt{5}}{2} \end{cases} \Rightarrow$

$\frac{7-3\sqrt{5}}{2} \leq x < 2$

$$42. \log_2 \frac{x + \sqrt{x^2 + 9}}{2x} > 1$$

$$\log_2 \frac{x + \sqrt{x^2 + 9}}{2x} > \log_2 2 \Rightarrow \frac{x + \sqrt{x^2 + 9}}{2x} > 2$$

$$\begin{cases} \frac{x + \sqrt{x^2 + 9}}{2x} > 0 \\ \frac{x + \sqrt{x^2 + 9}}{2x} > 2 \end{cases} \Rightarrow \frac{x + \sqrt{x^2 + 9}}{2x} > 2 \Rightarrow \frac{\sqrt{x^2 + 9} - 3x}{2x} > 0$$

$$N > 0: \sqrt{x^2 + 9} > 3x \Rightarrow \begin{cases} x^2 + 9 > 9x^2 \\ 3x \geq 0 \end{cases} \vee 3x < 0 \Rightarrow x < \frac{3}{2\sqrt{2}}$$

$$D > 0: x > 0$$

$$0 < x < \frac{3}{2\sqrt{2}}$$

$$43. 3 \log_2 x - \frac{12}{\log_2 x} < 5$$

$$c.a.: \begin{cases} x > 0 \\ \log_2 x \neq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \neq 1 \end{cases} \quad \text{Pongo: } \log_2 x = t$$

$$3t - \frac{12}{t} < 5 \Rightarrow \frac{3t^2 - 5t - 12}{t} < 0 \Rightarrow t < -\frac{4}{3} \vee 0 < t < 3$$

$$\log_2 x < -\frac{4}{3} \vee 0 < \log_2 x < 3 \Rightarrow x < \frac{1}{2} \sqrt[3]{\frac{1}{2}} \vee 1 < x < 8$$

$$\begin{cases} x < \frac{1}{2} \sqrt[3]{\frac{1}{2}} \vee 1 < x < 8 \\ x > 0 \\ x \neq 1 \end{cases} \Rightarrow 0 < x < \frac{1}{2} \sqrt[3]{\frac{1}{2}} \vee 1 < x < 8$$