

$$44. \frac{\log_2^2 x - 5 \log_2 x + 6}{1 - \log_2^2 x} \geq 0$$

$$c.a.: \begin{cases} x > 0 \\ \log_2 x \neq \pm 1 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \neq 2 \wedge x \neq \frac{1}{2} \end{cases} \quad \text{Pongo: } \log_2 x = t$$

$$\frac{t^2 - 5t + 6}{1 - t^2} \geq 0 \Rightarrow \frac{(t-2)(t-3)}{(1-t)(1+t)} \geq 0 \Rightarrow$$

$$-1 < t < 1 \vee 2 \leq t \leq 3 \Rightarrow -1 < \log_2 x < 1 \vee 2 \leq \log_2 x \leq 3$$

$$\frac{1}{2} < x < 2 \vee 4 \leq x \leq 8$$

$$\begin{cases} x \neq \frac{1}{2} \wedge x \neq 2 \\ x > 0 \\ \frac{1}{2} < x < 2 \vee 4 \leq x \leq 8 \end{cases} \Rightarrow \frac{1}{2} < x < 2 \vee 4 \leq x \leq 8$$

$$45. (\log_2 |x|)^2 + 2 \log_2 |x| - 3 < 0$$

$$c.a.: x \neq 0 \quad \text{Pongo: } \log_2 |x| = t \Rightarrow t^2 + 2t - 3 < 0 \Rightarrow -3 < t < 1 \Rightarrow$$

$$-3 < \log_2 |x| < 1 \Rightarrow \frac{1}{8} < |x| < 2 \Rightarrow -2 < x < -\frac{1}{8} \vee \frac{1}{8} < x < 2$$

$$46. \log_3 |x| \geq -1$$

$$\log_3 |x| \geq -1 \Rightarrow \begin{cases} x \neq 0 \\ |x| \geq \frac{1}{3} \end{cases} \Rightarrow x \leq -\frac{1}{3} \vee x \geq \frac{1}{3}$$

$$47. \log_{\frac{1}{2}} (4x + x^2) \leq 1$$

$$\begin{cases} 4x + x^2 > 0 \\ 4x + x^2 \geq \frac{1}{2} \end{cases} \Rightarrow 4x + x^2 \geq \frac{1}{2} \Rightarrow 2x^2 + 8x - 1 \geq 0 \Rightarrow$$

$$x \leq \frac{-4 - 3\sqrt{2}}{2} \vee x \geq \frac{-4 + 3\sqrt{2}}{2}$$

$$48. \log_{\frac{1}{2}} \sqrt{x} \leq 4$$

$$\frac{1}{2} \log_{\frac{1}{2}} x \leq 4 \Rightarrow \log_{\frac{1}{2}} x \leq 8 \Rightarrow \begin{cases} x > 0 \\ x \geq \frac{1}{256} \end{cases} \Rightarrow x \geq \frac{1}{256}$$

49. $\log_{\frac{1}{3}}(2-x) - \log_{\frac{1}{3}}(1-2x) > 0$

$$\log_{\frac{1}{3}}(2-x) > \log_{\frac{1}{3}}(1-2x) \Rightarrow \begin{cases} 2-x > 0 \\ 1-2x > 0 \\ 2-x < 1-2x \end{cases} \Rightarrow \begin{cases} x < 2 \\ x < \frac{1}{2} \\ x < -1 \end{cases} \Rightarrow x < -1$$

50. $\log_2(e^{2x} - e^x) > 1$

$$\begin{cases} e^{2x} - e^x > 0 \\ e^{2x} - e^x > 2 \end{cases} \Rightarrow \begin{cases} e^x(e^x - 1) > 0 \\ (e^x - 2)(e^x + 1) > 0 \end{cases} \Rightarrow \begin{cases} e^x - 1 > 0 \\ e^x - 2 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > \ln 2 \end{cases} \Rightarrow x > \ln 2$$

51. $\ln(e^{2x} - 3 \cdot e^x + 2) \leq 0$

$$\begin{cases} e^{2x} - 3 \cdot e^x + 2 > 0 \\ e^{2x} - 3 \cdot e^x + 2 \leq 1 \end{cases} \Rightarrow \begin{cases} (e^x - 2)(e^x - 1) > 0 \\ e^{2x} - 3 \cdot e^x + 1 \leq 0 \end{cases} \Rightarrow$$

$$\begin{cases} e^x < 1 \vee e^x > 2 \\ \frac{3 - \sqrt{5}}{2} \leq e^x \leq \frac{3 + \sqrt{5}}{2} \end{cases} \Rightarrow \frac{3 - \sqrt{5}}{2} \leq e^x < 1 \vee 2 < e^x \leq \frac{3 + \sqrt{5}}{2}$$

$$\ln \frac{3 - \sqrt{5}}{2} \leq x < 0 \vee \ln 2 < x \leq \ln \frac{3 + \sqrt{5}}{2}$$

52. $\ln^4 x - 5 \ln^2 x + 4 \geq 0$

c.a.: $x > 0$: Pongo: $\ln x = t \Rightarrow t^4 - 5t^2 + 4 \geq 0 \Rightarrow t_{1,2}^2 = \frac{5 \pm \sqrt{25 - 16}}{2}$

$$t \leq -2 \vee -1 \leq t \leq 1 \vee t \geq 2 \Rightarrow \ln x \leq -2 \vee -1 \leq \ln x \leq 1 \vee \ln x \geq 2$$

$$\begin{cases} x > 0 \\ x \leq \frac{1}{e^2} \vee \frac{1}{e} \leq x \leq e \vee x \geq e^2 \end{cases} \Rightarrow 0 < x \leq \frac{1}{e^2} \vee \frac{1}{e} \leq x \leq e \vee x \geq e$$

53. $|\log_2 x - 4| > 5$

c.a.: $x > 0$: $\log_2 x - 4 < -5 \vee \log_2 x - 4 > 5$

$$\log_2 x < -1 \vee \log_2 x > 9 \Rightarrow x < \frac{1}{2} \vee x > 2^9$$

$$\begin{cases} x > 0 \\ x < \frac{1}{2} \vee x > 2^9 \end{cases} \Rightarrow 0 < x < \frac{1}{2} \vee x > 2^9$$

54. $\ln |x| \leq \ln |2 + x|$

c.a.: $x \neq 0 \wedge x \neq -2: |x| \leq |2 + x|$

$$\begin{cases} x < -2 \\ -x \leq -2 - x \end{cases} \text{ imp.}$$

$$\begin{cases} -2 \leq x < 0 \\ -x \leq 2 + x \end{cases} \Rightarrow \begin{cases} -2 \leq x < 0 \\ 2x \geq -2 \end{cases} \Rightarrow \begin{cases} -2 \leq x < 0 \\ x \geq -1 \end{cases} \Rightarrow -1 \leq x < 0$$

$$\begin{cases} x \geq 0 \\ x \leq 2 + x \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ \forall x \end{cases} \Rightarrow x \geq 0 \qquad x \geq -1$$

$$\begin{cases} x \geq -1 \\ x \neq 0 \wedge x \neq -2 \end{cases} \Rightarrow x \geq -1 \wedge x \neq 0$$

55. $3 - \log_2 |x - 1| > 0$

c.a.: $x \neq 1: \log_2 |x - 1| < 3 \Rightarrow |x - 1| < 8 \Rightarrow -8 < x - 1 < 8$

$$\begin{cases} -7 < x < 9 \\ x \neq 1 \end{cases} \Rightarrow -7 < x < 1 \vee 1 < x < 9$$

56. $5 - \log_2 |x + 1| > 0$

c.a.: $x \neq -1: \log_2 |x + 1| < 5 \Rightarrow |x + 1| < 32 \Rightarrow -32 < x + 1 < 32$

$$\begin{cases} -33 < x < 31 \\ x \neq -1 \end{cases} \Rightarrow -33 < x < -1 \vee -1 < x < 31$$

57. $\log_2 |x| - \log_2 (x + 1) \leq 0$

c.a.: $\begin{cases} x \neq 0 \\ x + 1 > 0 \end{cases} \Rightarrow \begin{cases} x \neq 0 \\ x > -1 \end{cases}$

$\log_2 |x| \leq \log_2 (x + 1) \Rightarrow |x| \leq x + 1$

$$\begin{cases} x \geq 0 \\ x \leq x + 1 \end{cases} \vee \begin{cases} x < 0 \\ -x \leq x + 1 \end{cases} \Rightarrow x \geq 0 \vee -\frac{1}{2} \leq x < 0$$

$$\begin{cases} x \neq 0 \\ x > -1 \\ x \geq -\frac{1}{2} \end{cases} \Rightarrow -\frac{1}{2} \leq x < 0 \vee x > 0$$