

82.  $\log_{1/4} (x^2 + 2x + 1) < 2$

$$\begin{cases} x^2 + 2x + 1 > 0 \\ x^2 + 2x + 1 > \frac{1}{16} \end{cases} \Rightarrow \begin{cases} x \neq -1 \\ x < -\frac{5}{4} \vee x > -\frac{3}{4} \end{cases} \Rightarrow x < -\frac{5}{4} \vee x > -\frac{3}{4}$$

83.  $\log (x^2 - 1) - \log (x^2 - 7x + 12) < \log 4$

$$\log (x^2 - 1) < \log (x^2 - 7x + 12) + \log 4$$

$$\log (x^2 - 1) < \log 4 (x^2 - 7x + 12)$$

$$\begin{cases} x^2 - 1 > 0 \\ x^2 - 7x + 12 > 0 \\ x^2 - 1 < 4x^2 - 28x + 48 \end{cases} \Rightarrow \begin{cases} x < -1 \vee x > 1 \\ x < 3 \vee x > 4 \\ x < \frac{7}{3} \vee x > 7 \end{cases} \Rightarrow$$

$$x < -1 \vee 1 < x < \frac{7}{3} \vee x > 7$$

84.  $2 \log x - 1 > \log \left( x - \frac{5}{2} \right)$

$$2 \log x > 1 + \log \left( x - \frac{5}{2} \right)$$

$$\log x^2 > \log 10 \left( x - \frac{5}{2} \right)$$

$$\begin{cases} x > 0 \\ x - \frac{5}{2} > 0 \\ x^2 > 10x - 25 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > \frac{5}{2} \\ x \neq 5 \end{cases} \Rightarrow x > \frac{5}{2} \wedge x \neq 5$$

85.  $\log (5 - x) + \log (25 + 5x + x^2) < 3 \log (5 - x)$

$$\log (5 - x) (25 + 5x + x^2) < \log (5 - x)^3$$

$$\log (125 - x^3) < \log (125 - 75x + 15x^2 - x^3)$$

$$\begin{cases} 5 - x > 0 \\ 25 + 5x + x^2 > 0 \\ 125 - x^3 < 125 - 75x + 15x^2 - x^3 \end{cases} \Rightarrow \begin{cases} x < 5 \\ \forall x \\ x < 0 \vee x > 5 \end{cases} \Rightarrow x < 0$$

$$86. \quad 4 \log \frac{x}{2} + 3 \log \frac{x}{3} > 5 \log x - \log 12$$

$$4 \log \frac{x}{2} + 3 \log \frac{x}{3} - 5 \log x + \log 12 > 0$$

$$\log 12 \left( \frac{x^4}{16} \cdot \frac{x^3}{27} : x^5 \right) > 0 \quad \Rightarrow \quad \log \frac{x^2}{36} > \log 1$$

$$\begin{cases} x > 0 \\ \frac{x^2}{36} > 1 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x < -6 \vee x > 6 \end{cases} \Rightarrow \boxed{x > 6}$$

$$87. \quad \frac{\log_3 (1-x) - 1}{\log_3 (x^2 - 7) - 2} > 1$$

$$\frac{\log_3 (1-x) - 1 - \log_3 (x^2 - 7) + 2}{\log_3 (x^2 - 7) - 2} > 0$$

$$\text{c.a.: } \begin{cases} 1-x > 0 \\ x^2 - 7 > 0 \end{cases} \Rightarrow \begin{cases} x < 1 \\ x < -\sqrt{7} \vee x > \sqrt{7} \end{cases} \Rightarrow x < -\sqrt{7}$$

$$N > 0: \log_3 (1-x) + 1 > \log_3 (x^2 - 7)$$

$$3 - 3x > x^2 - 7 \Rightarrow x^2 + 3x - 10 < 0 \Rightarrow -5 < x < 2$$

$$D > 0: \log_3 (x^2 - 7) > 2 \Rightarrow x^2 - 7 > 9 \Rightarrow x < -4 \vee x > 4$$

$$\begin{cases} -5 < x < -4 \vee 2 < x < 4 \\ x < -\sqrt{7} \end{cases} \Rightarrow \boxed{-5 < x < -4}$$

$$88. \quad \frac{1-2^x}{1-\log_2 x} > 0$$

$$\text{c.a.: } x > 0 \quad N > 0: 1 - 2^x > 0 \Rightarrow 2^x < 1 \Rightarrow x < 0$$

$$D > 0: 1 - \log_2 x > 0 \Rightarrow \log_2 x < 1 \Rightarrow x < 2$$

$$\begin{cases} x > 0 \\ x < 0 \vee x > 2 \end{cases} \Rightarrow \boxed{x > 2}$$

$$89. \quad \log x \log (x-5) > 0$$

$$I \ F > 0: \log x > 0 \Rightarrow x > 1$$

$$II \ F > 0: \log (x-5) > 0 \Rightarrow x-5 > 1 \Rightarrow x > 6$$

$$\begin{cases} x < 1 \vee x > 6 \\ x > 0 \\ x - 5 > 0 \end{cases} \Rightarrow \boxed{x > 6}$$

90.  $\log(x^2 - 7) < 2 \log(x + 3)$

$$\log(x^2 - 7) < \log(x + 3)^2$$

$$\begin{cases} x^2 - 7 > 0 \\ x + 3 > 0 \\ x^2 - 7 < x^2 + 6x + 9 \end{cases} \Rightarrow \begin{cases} x < -\sqrt{7} \vee x > \sqrt{7} \\ x > -3 \\ x > -\frac{8}{3} \end{cases} \Rightarrow -\frac{8}{3} < x < -\sqrt{7} \vee x > \sqrt{7}$$

91.  $\log_2 [\log_2(x + 3)] > 0$

$$\log_2 [\log_2(x + 3)] > \log_2 1$$

$$\log_2(x + 3) > 1$$

$$\log_2(x + 3) > \log_2 2$$

$$x + 3 > 2 \Rightarrow x > -1$$

$$\begin{cases} x + 3 > 0 \\ \log_2(x + 3) > 0 \\ x > -1 \end{cases} \Rightarrow \begin{cases} x > -3 \\ x > -2 \\ x > -1 \end{cases} \Rightarrow x > -1$$

92.  $\log(x + 3) + \log(x - 7) - \log(1 - 2x) \geq 0$

$$\log(x + 3)(x - 7) \geq \log(1 - 2x)$$

$$\begin{cases} x + 3 > 0 \\ x - 7 > 0 \\ 1 - 2x > 0 \\ x^2 - 4x - 21 \geq 1 - 2x \end{cases} \Rightarrow \begin{cases} x > -3 \\ x > 7 \\ x < \frac{1}{2} \\ x^2 - 2x - 22 \geq 0 \end{cases} \Rightarrow$$

$$\begin{cases} x > -3 \\ x > 7 \\ x < \frac{1}{2} \\ x \leq 1 - \sqrt{23} \vee x \geq 1 + \sqrt{23} \end{cases} \Rightarrow \text{imp.}$$

93.  $\log(x + 2) > 1$

$$\begin{cases} x + 2 > 0 \\ x + 2 > 10 \end{cases} \Rightarrow \begin{cases} x > -2 \\ x > 8 \end{cases} \Rightarrow x > 8$$