1. $sen^2 x - sen\, x = 0$

$sen\, x\,(sen\, x - 1) = 0 \qquad \left| \begin{array}{l} sen\, x = 0 \\ sen\, x = 1 \end{array} \right. \qquad \boxed{\begin{array}{l} x = k\pi \\ x = \dfrac{\pi}{2} + 2\,k\pi \end{array}}$

2. $2\, sen^2 x - 1 = 0$

$sen^2 x = \dfrac{1}{2} \qquad \left| \begin{array}{l} sen\, x = -\dfrac{\sqrt{2}}{2} \\ sen\, x = \dfrac{\sqrt{2}}{2} \end{array} \right. \qquad \begin{array}{l} x = \dfrac{5}{4}\pi + 2k\pi;\; x = \dfrac{7}{4}\pi + 2k\pi \\ x = \dfrac{\pi}{4} + 2k\pi;\; x = \dfrac{3}{4}\pi + 2k\pi \end{array} \qquad \boxed{x = \dfrac{\pi}{4} + k\dfrac{\pi}{2}}$

3. $\cos^2 x + \cos x = 0$

$\cos x\,(\cos x + 1) = 0 \qquad \left| \begin{array}{l} \cos x = 0 \\ \cos x = -1 \end{array} \right. \qquad \boxed{\begin{array}{l} x = \dfrac{\pi}{2} + k\pi \\ x = \pi + 2\,k\pi \end{array}}$

4. $tg^2 x = 3$

$\left| \begin{array}{l} tg\, x = -\sqrt{3} \\ tg\, x = \sqrt{3} \end{array} \right. \qquad \begin{array}{l} x = \dfrac{2}{3}\pi + k\pi \\ x = \dfrac{\pi}{3} + k\pi \end{array} \qquad \boxed{x = \pm\dfrac{\pi}{3} + k\pi}$

5. $tg^2 x - \sqrt{3}\, tg\, x = 0$

$tg\, x\,(tg\, x - \sqrt{3}) = 0 \qquad \left| \begin{array}{l} tg\, x = 0 \\ tg\, x = \sqrt{3} \end{array} \right. \qquad \boxed{\begin{array}{l} x = k\pi \\ x = \dfrac{\pi}{3} + k\pi \end{array}}$

6. $ctg^2\left(x + \dfrac{\pi}{4}\right) + \sqrt{3}\, ctg\left(x + \dfrac{\pi}{4}\right) = 0$

Pongo $x + \dfrac{\pi}{4} = t$ e l'equazione diventa: $ctg^2 t + \sqrt{3}\, ctg\, t = 0$

$ctg\, t\,(ctg\, t + \sqrt{3}) = 0 \qquad \left| \begin{array}{lll} ctg\, t = 0 & t = \dfrac{\pi}{2} + k\pi & x + \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi \\ ctg\, t = -\sqrt{3} & t = \dfrac{5}{6}\pi + k\pi & x + \dfrac{\pi}{4} = \dfrac{5}{6}\pi + k\pi \end{array} \right. \qquad \boxed{\begin{array}{l} x = \dfrac{\pi}{4} + k\pi \\ x = \dfrac{7}{12}\pi + k\pi \end{array}}$

7. $sen^2 x - 3\, sen\, x + 2 = 0$

$sen\, x = \dfrac{3 \pm \sqrt{9-8}}{2} = \left\langle \begin{array}{l} 2 \\ 1 \end{array} \right. \qquad \begin{array}{l} sen\, x = 2 \quad imp. \\ sen\, x = 1 \end{array} \qquad \boxed{x = \dfrac{\pi}{2} + 2k\pi}$

8. $sen^2 x + sen\,x - \cos^2 x = 0$

$sen^2 x + sen\,x - (1 - sen^2 x) = 0$

$sen^2 x + sen\,x - 1 + sen^2 x = 0 \qquad 2\,sen^2 x + sen\,x - 1 = 0$

$$sen\,x = \frac{-1 \pm \sqrt{1+8}}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ -1 \end{array} \right. \qquad \begin{array}{l} sen\,x = \frac{1}{2} \\ sen\,x = -1 \end{array}$$

$x = \frac{\pi}{6} + 2k\pi; \quad x = \frac{5}{6}\pi + 2k\pi$

$x = \frac{3}{2}\pi + 2k\pi$

9. $2\cos^2 x - (2 + \sqrt{3})\cos x + \sqrt{3} = 0$

$$\cos x = \frac{2+\sqrt{3} \pm \sqrt{(2+\sqrt{3})^2 - 8\sqrt{3}}}{4} = \frac{2+\sqrt{3} \pm (2-\sqrt{3})}{4} = \left\langle \begin{array}{l} 1 \\ \frac{\sqrt{3}}{2} \end{array} \right. \qquad \begin{array}{l} \cos x = 1 \\ \cos x = \frac{\sqrt{3}}{2} \end{array}$$

$x = 2k\pi$

$x = \pm\frac{\pi}{6} + 2k\pi$

10. $ctg\left(x - \frac{\pi}{8}\right) + tg\left(x - \frac{\pi}{8}\right) - 2 = 0$

Pongo $x - \frac{\pi}{8} = t$ e l'equazione diventa: $ctg\,t + tg\,t - 2 = 0$

$\frac{1}{tg\,t} + tg\,t - 2 = 0 \qquad tg^2 t - 2tg\,t + 1 = 0 \qquad c.a.: tg\,t \neq 0$

$tg\,t = 1 \qquad t = \frac{\pi}{4} + k\pi \quad \Rightarrow \quad x - \frac{\pi}{8} = \frac{\pi}{4} + k\pi \quad \Rightarrow \quad x = \frac{3}{4}\pi + k\pi$

11. $\cos^3 x - 2\cos^2 x + 1 = 0$

Applicando la regola di Ruffini e dividendo per $\cos x - 1$ ottengo:

$(\cos x - 1)(\cos^2 x - \cos x - 1) = 0$

$\cos x = 1 \qquad x = 2k\pi$

$$\cos^2 x - \cos x - 1 = 0 \qquad \cos x = \frac{1 \pm \sqrt{1+4}}{2} = \left\langle \begin{array}{l} \frac{1+\sqrt{5}}{2} \quad \textit{non accettabile perché maggiore di 1} \\ \frac{1-\sqrt{5}}{2} \end{array} \right.$$

$x = arc\cos\frac{1-\sqrt{5}}{2}$

12. $tg^3 x - tg^2 x - 3\, tg\, x + 3 = 0$

Proseguendo con un raccoglimento parziale: $tg^2 x\,(tg\, x - 1) - 3\,(tg\, x - 1) = 0$

$(tg\, x - 1)\;(tg^2 x - 3) = 0$

$tg\, x = 1 \qquad x = \frac{\pi}{4} + k\pi$

$tg^2 x = 3 \qquad \begin{cases} tg\, x = \sqrt{3} & x = \frac{\pi}{3} + k\pi \\ tg\, x = -\sqrt{3} & x = \frac{2}{3}\pi + k\pi \end{cases} \qquad x = \pm\frac{\pi}{3} + k\pi$

13. $4\, sen\, 3x \cos 3x = \sqrt{3}$

Applicando la formula di duplicazione del seno: $2 \cdot 2\, sen\, 3x \cos 3x = \sqrt{3}$

$$2\, sen\, 6x = \sqrt{3}$$

$sen\, 6x = \frac{\sqrt{3}}{2} \qquad \begin{cases} 6x = \frac{\pi}{3} + 2k\pi \\ 6x = \frac{2}{3}\pi + 2k\pi \end{cases} \qquad \begin{matrix} x = \frac{\pi}{18} + k\frac{\pi}{3} \\ x = \frac{\pi}{9} + k\frac{\pi}{3} \end{matrix}$

14. $sen\, x - \cos x + 1 = 0$

Si tratta di un'equazione lineare, che posso risolvere graficamente ponendo: $sen\, x = Y \qquad \cos x = X$

$\begin{cases} Y - X + 1 = 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} Y = X - 1 \\ X^2 + X^2 - 2X + 1 = 1 \end{cases} \qquad \begin{cases} Y = X - 1 \\ 2X^2 - 2X = 0 \end{cases}$

$\begin{cases} X = 0 \\ Y = -1 \end{cases} \qquad x = \frac{3}{2}\pi + 2k\pi \qquad\qquad \begin{cases} X = 1 \\ Y = 0 \end{cases} \qquad x = 2k\pi$

15. $\sqrt{3} \cos\left(\frac{3}{2}\pi + x\right) + \cos(x - \pi) = 2$

Utilizzando le formule degli archi associati, l'equazione diventa: $\sqrt{3}\, sen\, x - \cos x = 2$

Si tratta di un'equazione lineare, che posso risolvere graficamente ponendo: $sen\, x = Y \qquad \cos x = X$

$\begin{cases} Y\sqrt{3} - X = 2 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = Y\sqrt{3} - 2 \\ 3Y^2 - 4\sqrt{3}Y + 4 + Y^2 = 1 \end{cases}$

$\begin{cases} X = Y\sqrt{3} - 2 \\ (2Y - \sqrt{3})^2 = 0 \end{cases} \qquad \begin{cases} X = -\frac{1}{2} \\ Y = \frac{\sqrt{3}}{2} \end{cases} \qquad x = \frac{2}{3}\pi + 2k\pi$