EQUAZIONI NUMERICHE FRAZIONARIE

1. $\dfrac{2x+1}{x} = \dfrac{4x+1}{2x-1}$ $\qquad\qquad\qquad\qquad$ $c.a.:\ x \neq 0\ \wedge\ x \neq \dfrac{1}{2}$

$\dfrac{(2x+1)\,(2x-1)}{x\,(2x-1)} = \dfrac{x\,(4x+1)}{x\,(2x-1)}$

$4x^2 - 1 = 4x^2 + x$ $\qquad\qquad\qquad$ $\boxed{x = -1}$

2. $\dfrac{x-1}{x+2} - \dfrac{4}{3} = \dfrac{1}{2}$ $\qquad\qquad\qquad\qquad$ $c.a.:\ x \neq -2$

$\dfrac{6\,(x-1) - 8\,(x+2)}{6\,(x+2)} = \dfrac{3\,(x+2)}{6\,(x+2)}$

$6x - 6 - 8x - 16 = 3x + 6$ $\qquad\qquad$ $-5x = 28$ $\qquad\qquad$ $\boxed{x = -\dfrac{28}{5}}$

3. $\dfrac{x-2}{x-1} - 2 = \dfrac{2}{x-1}$ $\qquad\qquad\qquad\qquad$ $c.a.:\ x \neq 1$

$\dfrac{x - 2 - 2\,(x-1)}{x-1} = \dfrac{2}{x-1}$

$x - 2 - 2x + 2 = 2$ $\qquad\qquad$ $-x = 2$ $\qquad\qquad$ $\boxed{x = -2}$

4. $\dfrac{2}{x-1} - \dfrac{1}{x} = \dfrac{1}{x^2 - x}$ $\qquad\qquad$ $\dfrac{2}{x-1} - \dfrac{1}{x} = \dfrac{1}{x\,(x-1)}$ $\qquad$ $c.a.:\ x \neq 1\ \wedge\ x \neq 0$

$\dfrac{2x - x + 1 = 1}{x\,(x-1)}$ $\qquad\qquad$ $x = 0\ non\ acc.\ per\ c.a.$ $\qquad$ $\boxed{imp.}$

5. $\dfrac{2x+5}{x-3} = \dfrac{1}{2} + \dfrac{x}{2x-6}$ $\qquad\qquad$ $\dfrac{2x+5}{x-3} = \dfrac{1}{2} + \dfrac{x}{2\,(x-3)}$

$\dfrac{4x + 10 = x - 3 + x}{2\,(x-3)}$ $\qquad\qquad\qquad$ $c.a.:\ x \neq 3$

$2x = -13$ $\qquad\qquad$ $\boxed{x = -\dfrac{13}{2}}$

6. $\dfrac{1}{2x-1} + \dfrac{1}{2x+2} - \dfrac{1}{x} = 0$ $\qquad\qquad$ $\dfrac{1}{2x-1} + \dfrac{1}{2\,(x+1)} - \dfrac{1}{x} = 0$

$\dfrac{2x\,(x+1) + x\,(2x-1) - 2\,(x+1)\,(2x-1)}{2\,x\,(x+1)\,(2x-1)} = 0$ $\qquad$ $c.a.:\ x \neq 0\ \wedge\ x \neq -1\ \wedge\ x \neq \dfrac{1}{2}$

$2x^2 + 2x + 2x^2 - x - 4x^2 + 2x - 4x + 2 = 0$ $\qquad\qquad$ $-x = -2$ $\qquad$ $\boxed{x = 2}$

7. $\dfrac{2\,(x+7)}{x^2-4} + \dfrac{x+4}{x+2} = \dfrac{x+3}{x-2}$

$\dfrac{2x+14}{(x-2)\,(x+2)} + \dfrac{x+4}{x+2} = \dfrac{x+3}{x-2}$

$\dfrac{2x+14+(x+4)\,(x-2)=(x+3)\,(x+2)}{(x-2)\,(x+2)}$ $\qquad$ $c.a.: x \neq 2 \ \wedge \ x \neq -2$

$2x+14+x^2-2x+4x-8 = x^2+2x+3x+6$ $\qquad$ $\boxed{x=0}$

8. $\dfrac{4x-3}{4x^2-9} + \dfrac{2}{2x+3} = \dfrac{1}{2x+3} - \dfrac{4}{6x+9}$

$\dfrac{4x-3}{(2x-3)\,(2x+3)} + \dfrac{2}{2x+3} - \dfrac{1}{2x+3} + \dfrac{4}{3\,(2x+3)} = 0$

$\dfrac{4x-3}{(2x-3)\,(2x+3)} + \dfrac{1}{2x+3} + \dfrac{4}{3\,(2x+3)} = 0$

$\dfrac{3\,(4x-3)+3\,(2x-3)+4\,(2x-3)}{3\,(2x-3)\,(2x+3)} = 0$ $\qquad$ $c.a.: x \neq \pm\dfrac{3}{2}$

$12x-9+6x-9+8x-12 = 0$ $\qquad$ $\boxed{x=\dfrac{15}{13}}$

9. $\dfrac{x+2}{x} = \dfrac{3}{x} - \dfrac{1}{5x}$ $\qquad\qquad\qquad\qquad$ $c.a.: x \neq 0$

$\dfrac{5\,(x+2)}{5x} = \dfrac{15-1}{5x}$

$5x+10 = 15-1$ $\qquad$ $5x=4$ $\qquad$ $\boxed{x=\dfrac{4}{5}}$

10. $\dfrac{2}{3} - \dfrac{4}{x+1} = \dfrac{x-3}{x+1}$ $\qquad\qquad\qquad\qquad$ $c.a.: x \neq -1$

$\dfrac{2\,(x+1)-12}{3\,(x+1)} = \dfrac{3\,(x-3)}{3\,(x+1)}$

$2x+2-12 = 3x-9$ $\qquad$ $-x=1$ $\qquad$ $x=-1$ $\ non\ acc.\ per\ c.a.$ $\qquad$ $\boxed{imp.}$

11. $\dfrac{x-1}{2x+6} + \dfrac{x+2}{x+3} = \dfrac{3}{4}$ $\qquad\qquad\qquad\qquad$ $c.a.: x \neq -3$

$\dfrac{2x-2+4x+8=3x+9}{4\,(x+3)}$ $\qquad$ $3x=3$ $\qquad$ $\boxed{x=1}$

12. $\dfrac{2x-3}{x^2+1} - \dfrac{4}{5} = \dfrac{8x-(2x+1)^2}{5x^2+5}$ $\dfrac{10x-15-4x^2-4 = 8x-4x^2-4x-1}{5\,(x^2+1)}$

$6x = 18$ $\boxed{x=3}$

13. $\dfrac{5}{2x^2+x} - \dfrac{3}{2x^2-x} = \dfrac{20}{4x^2-1}$

$\dfrac{5}{x\,(2x+1)} - \dfrac{3}{x\,(2x-1)} = \dfrac{20}{(2x+1)\,(2x-1)}$ $c.a.: x \neq 0 \,\wedge\, x \neq \pm\dfrac{1}{2}$

$\dfrac{5\,(2x-1) - 3\,(2x+1)}{x\,(2x+1)\,(2x-1)} = \dfrac{20\,x}{x\,(2x+1)\,(2x-1)}$

$10x - 5 - 6x - 3 = 20x$ $-16\,x = 8$

$x = -\dfrac{1}{2}$ *non acc. per c.a.*   *imp.*

14. $\dfrac{3}{x+2} - \dfrac{x+1}{x^2+2x} = \dfrac{2x-1}{x^2+4x+4}$ $\dfrac{3}{x+2} - \dfrac{x+1}{x\,(x+2)} = \dfrac{2x-1}{(x+2)^2}$

$\dfrac{3x\,(x+2) - (x+1)\,(x+2)}{x\,(x+2)^2} = \dfrac{2x^2-x}{x\,(x+2)^2}$ $c.a.: x \neq 0 \,\wedge\, x \neq -2$

$3x^2 + 6x - x^2 - 2x - x - 2 - 2x^2 + x = 0$

$4x = 2$ $\boxed{x = \dfrac{1}{2}}$

15. $\dfrac{x+1}{x} + \dfrac{1+x}{x+2} = \dfrac{2}{x^2+2x} + 2$

$\dfrac{(x+2)\,(x+1) + x\,(1+x) - 2 - 2x\,(x+2)}{x\,(x+2)} = 0$ $c.a.: x \neq 0 \,\wedge\, x \neq -2$

$x^2 + x + 2x + 2 + x + x^2 - 2 - 2x^2 - 4x = 0$ *ind. ma con* $x \neq 0 \,\wedge\, x \neq -2$

16. $1 - \dfrac{1-2x}{4x^2-6x} = \dfrac{2x}{2x-3} - \dfrac{1}{2x}$ $1 - \dfrac{1-2x}{2x\,(2x-3)} = \dfrac{2x}{2x-3} - \dfrac{1}{2x}$

$\dfrac{4x^2-6x-1+2x}{2x\,(2x-3)} = \dfrac{4x^2-2x+3}{2x\,(2x-3)}$ $c.a.: x \neq 0 \,\wedge\, x \neq \dfrac{3}{2}$

$-2x = 4$ $\boxed{x = -2}$

17. $\dfrac{2}{x^2-1} + \dfrac{1}{x^2-x-2} = \dfrac{3}{x^2-3x+2}$

$\dfrac{2}{(x-1)(x+1)} + \dfrac{1}{(x-2)(x+1)} = \dfrac{3}{(x-2)(x-1)}$

$\dfrac{2(x-2)+x-1-3(x+1)}{(x-1)(x+1)(x-2)} = 0$ $\qquad\qquad$ c.a.: $x \neq 2 \ \wedge \ x \neq \pm 1$

$2x-4+x-1-3x-3 = 0$

$0x = 8$ $\qquad\qquad\qquad$ *imp.*

18. $x^2 - \dfrac{9(x-3)}{4x-6} = \dfrac{2x^3}{2x-3} + \dfrac{6x^2-9x}{6-4x}$

$x^2 - \dfrac{9(x-3)}{2(2x-3)} = \dfrac{2x^3}{2x-3} + \dfrac{6x^2-9x}{-2(2x-3)}$

$x^2 - \dfrac{9(x-3)}{2(2x-3)} = \dfrac{2x^3}{2x-3} - \dfrac{6x^2-9x}{2(2x-3)}$

$\dfrac{2x^2(2x-3)-9(x-3)}{2(2x-3)} = \dfrac{4x^3-(6x^2-9x)}{2(2x-3)}$ $\qquad\qquad$ c.a.: $x \neq \dfrac{3}{2}$

$4x^3-6x^2-9x+27 = 4x^3-6x^2+9x$

$-18x = -27$ $\qquad\qquad\qquad x = \dfrac{3}{2}$ *non acc. per c.a.* $\Rightarrow$ *imp.*

19. $\left(\dfrac{1}{x+4} - \dfrac{1}{5} \right) : \left(\dfrac{1}{x-4} + \dfrac{1}{3} \right) = 1$

$\dfrac{5-(x+4)}{5(x+4)} : \dfrac{3+x-4}{3(x-4)} = 1$

$\dfrac{5-x-4}{5(x+4)} \cdot \dfrac{3(x-4)}{x-1} = 1$ $\qquad\qquad\qquad \dfrac{1-x}{5(x+4)} \cdot \dfrac{3(x-4)}{x-1} = 1$

$\dfrac{-(x-1)}{5(x+4)} \cdot \dfrac{3(x-4)}{x-1} = 1$ $\qquad\qquad\qquad \dfrac{-3(x-4)}{5(x+4)} = 1$

$\dfrac{-3(x-4)}{5(x+4)} = \dfrac{5(x+4)}{5(x+4)}$ $\qquad\qquad$ c.a.: $x \neq -4 \ \wedge \ x \neq 1$

$-3x+12 = 5x+20$ $\qquad\qquad\qquad x = -1$

20. $\left(\dfrac{x}{12} - \dfrac{12}{x}\right) : \left(1 + \dfrac{12}{x}\right) = \dfrac{x-12}{12}$

$\dfrac{x^2 - 144}{12\,x} : \dfrac{x+12}{x} = \dfrac{x-12}{12}$ $\qquad\qquad\qquad \dfrac{(x-12)\,(x+12)}{12\,x} \cdot \dfrac{x}{x+12} = \dfrac{x-12}{12}$

$\dfrac{x-12}{12} = \dfrac{x-12}{12}$ $\qquad\qquad\qquad c.a.:\ x \neq 0\ \wedge\ x \neq -12$

$\boxed{ind:\ x \neq 0\ \wedge\ x \neq -12}$

21. $\dfrac{1}{x}\left(\dfrac{1}{x-1} - 2\right) + 2\left(\dfrac{2}{x+1} - \dfrac{1}{x}\right) + \dfrac{4}{3x^2 - 3} = 0$

$\dfrac{1}{x} \cdot \dfrac{1 - 2\,(x-1)}{x-1} + 2 \cdot \dfrac{2x - (x+1)}{x\,(x+1)} + \dfrac{4}{3\,(x^2 - 1)} = 0$

$\dfrac{1 - 2x + 2}{x\,(x-1)} + 2 \cdot \dfrac{2x - x - 1}{x\,(x+1)} + \dfrac{4}{3\,(x-1)\,(x+1)} = 0$

$\dfrac{3 - 2x}{x\,(x-1)} + \dfrac{2x - 2}{x\,(x+1)} + \dfrac{4}{3\,(x-1)\,(x+1)} = 0$

$\dfrac{3\,(3 - 2x)\,(x+1) + 6\,(x-1)^2 + 4x}{3x\,(x-1)\,(x+1)} = 0$ $\qquad\qquad c.a.:\ x \neq \pm 1\ \wedge\ x \neq 0$

$9x + 9 - 6x^2 - 6x + 6x^2 - 12x + 6 + 4x = 0$

$-5x = -15$ $\qquad\qquad\qquad \boxed{x = 3}$

22. $x\left(\dfrac{1}{x-2} + \dfrac{1}{1-x}\right) - (x-2)\left(\dfrac{1}{x-1} - \dfrac{1}{x}\right) = \dfrac{4}{x^2 - 2x}$

$x\,\dfrac{1 - x + x - 2}{(x-2)\,(1-x)} - (x-2)\,\dfrac{x - (x-1)}{x\,(x-1)} = \dfrac{4}{x\,(x-2)}$

$-\dfrac{x}{(x-2)\,(1-x)} - \dfrac{x-2}{x\,(x-1)} = \dfrac{4}{x\,(x-2)}$

$\dfrac{x}{(x-2)\,(x-1)} - \dfrac{x-2}{x\,(x-1)} = \dfrac{4}{x\,(x-2)}$

$\dfrac{x^2 - (x-2)^2}{x\,(x-2)\,(x-1)} = \dfrac{4\,(x-1)}{x\,(x-2)\,(x-1)}$ $\qquad\qquad c.a.:\ x \neq 1\ \wedge\ x \neq 2\ \wedge\ x \neq 0$

$x^2 - x^2 + 4x - 4 = 4x - 4$ $\qquad\qquad \boxed{ind:\ x \neq 1\ \wedge\ x \neq 2\ \wedge\ x \neq 0}$