

PRODOTTI E QUOZIENTI DI RADICALI IN \mathbb{R}_0^+

$$1. \quad \sqrt{2} \cdot \sqrt{8} = 2^{\frac{1}{2}} \cdot 2^{\frac{3}{2}} = 2^{\frac{1}{2} + \frac{3}{2}} = 2^2 = 4$$

$$2. \quad \sqrt{3} \cdot \sqrt{27} = 3^{\frac{1}{2}} \cdot 3^{\frac{3}{2}} = 3^{\frac{1}{2} + \frac{3}{2}} = 3^2 = 9$$

$$3. \quad \sqrt{12} \cdot \sqrt{3} = 2^{\frac{2}{2}} \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 2 \cdot 3^{\frac{1}{2} + \frac{1}{2}} = 2 \cdot 3 = 6$$

$$4. \quad \sqrt{6} \cdot \sqrt{18} \cdot \sqrt{12} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{2}{2}} \cdot 2^{\frac{2}{2}} \cdot 3^{\frac{1}{2}} = 2^{\frac{1}{2} + \frac{1}{2} + 1} \cdot 3^{\frac{1}{2} + 1 + \frac{1}{2}} = 2^2 \cdot 3^2 = 6^2 = 36$$

$$5. \quad \sqrt{6} \cdot \sqrt{5} \cdot \sqrt{30} = 6^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 6^{\frac{1}{2}} = 6^{\frac{1}{2} + \frac{1}{2}} \cdot 5^{\frac{1}{2} + \frac{1}{2}} = 6 \cdot 5 = 30$$

$$6. \quad \sqrt{125} : \sqrt{5} = 5^{\frac{3}{2}} : 5^{\frac{1}{2}} = 5^{\frac{3}{2} - \frac{1}{2}} = 5^{\frac{2}{2}} = 5$$

$$7. \quad \sqrt{20} : \sqrt{45} = \frac{2^{\frac{2}{2}} \cdot 5^{\frac{1}{2}}}{3^{\frac{2}{2}} \cdot 5^{\frac{2}{2}}} = \frac{2}{3}$$

$$\sqrt{72} : \sqrt{32} = \frac{2^{\frac{3}{2}} \cdot 3^{\frac{2}{2}}}{2^{\frac{5}{2}}} = \frac{3}{2^{\frac{5}{2} - \frac{3}{2}}} = \frac{3}{2}$$

$$8. \quad \frac{\sqrt{15}}{\sqrt{3}} \cdot \sqrt{125} = \frac{5^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}} \cdot 5^{\frac{3}{2}} = 5^{\frac{1}{2} + \frac{3}{2}} = 5^2 = 25$$

$$9. \quad \sqrt{\frac{7}{3}} \cdot \sqrt{\frac{1}{14}} \cdot \sqrt{\frac{2}{3}} = \frac{7^{\frac{1}{2}}}{3^{\frac{1}{2}}} \cdot \frac{1}{2^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}} \cdot \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}} = \frac{1}{3}$$

$$10. \quad \sqrt{\frac{10}{3}} \cdot \frac{\sqrt{6}}{\sqrt{35}} \cdot \sqrt{7} = \frac{2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}}{3^{\frac{1}{2}}} \cdot \frac{2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{5^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}} \cdot 7^{\frac{1}{2}} = 2$$

$$11. \quad \sqrt[3]{3} \cdot \sqrt[3]{9} = 3^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} = 3^{\frac{1}{3} + \frac{2}{3}} = 3$$

$$\sqrt[3]{4} \cdot \sqrt[3]{9} \cdot \sqrt[3]{6} = 2^{\frac{2}{3}} \cdot 3^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} = 2 \cdot 3 = 6$$

$$12. \quad \sqrt[6]{50} \cdot \sqrt[6]{12} \cdot \sqrt[6]{45} = 2^{\frac{1}{6}} \cdot 5^{\frac{2}{6}} \cdot 2^{\frac{2}{6}} \cdot 3^{\frac{1}{6}} \cdot 5^{\frac{1}{6}} \cdot 3^{\frac{2}{6}} = 2^{\frac{1}{6} + \frac{2}{6}} \cdot 3^{\frac{1}{6} + \frac{2}{6}} \cdot 5^{\frac{2}{6} + \frac{1}{6}} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 30^{\frac{1}{2}} = \sqrt{30}$$

$$13. \quad \sqrt[6]{128} : (\sqrt[6]{8} \cdot \sqrt[6]{4}) = 2^{\frac{7}{6}} : (2^{\frac{3}{6}} \cdot 2^{\frac{2}{6}}) = 2^{\frac{7}{6}} : 2^{\frac{5}{6}} = 2^{\frac{2}{6}} = 2$$

$$14. \quad (\sqrt[6]{128} : \sqrt[6]{8}) : \sqrt[6]{4} = (2^{\frac{7}{6}} : 2^{\frac{3}{6}}) : 2^{\frac{2}{6}} = 2^{\frac{4}{6}} : 2^{\frac{2}{6}} = 2^{\frac{2}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$15. \quad \sqrt[6]{\frac{a^2 - ab}{b^2}} \cdot \sqrt[6]{\frac{ab^2 + b^3}{4a - 4b}} \cdot \sqrt[6]{\frac{a^2}{a^2 + ab}} = \left(\frac{a(a-b)}{b^2} \cdot \frac{b^2(a+b)}{4(a-b)} \cdot \frac{a^2}{a(a+b)} \right)^{\frac{1}{6}} = \frac{a^{\frac{2}{6}}}{2^{\frac{2}{6}}} = \sqrt[3]{\frac{a}{2}}$$

$$16. \quad \sqrt[3]{\frac{a^3 - b^3}{a^2 - b^2}} \cdot \sqrt[3]{\frac{a^2 + 2ab + b^2}{a+b}} \cdot \sqrt[3]{\frac{(a^2 + b^2)^3}{a^2 + ab + b^2}} =$$

$$17. \quad = \left(\frac{(a-b)(a^2 + ab + b^2)}{(a-b)(a+b)} \cdot \frac{(a+b)^2}{a+b} \cdot \frac{(a^2 + b^2)^3}{a^2 + ab + b^2} \right)^{\frac{1}{3}} = (a^2 + b^2)^{\frac{3}{3}} = a^2 + b^2$$

$$18. \quad \sqrt[4]{\frac{4x-4y}{9x}} : \sqrt{\frac{2x-2y}{3x}} = \frac{2^{\frac{2}{4}} (x-y)^{\frac{1}{4}}}{3^{\frac{2}{4}} x^{\frac{1}{4}}} \cdot \frac{3^{\frac{1}{2}} x^{\frac{1}{2}}}{2^{\frac{1}{2}} (x-y)^{\frac{1}{2}}} = \frac{x^{\frac{1}{2} - \frac{1}{4}}}{(x-y)^{\frac{1}{2} - \frac{1}{4}}} = \frac{x^{\frac{1}{4}}}{(x-y)^{\frac{1}{4}}} = \sqrt[4]{\frac{x}{x-y}}$$

19. $\sqrt{\frac{1-a}{1+a}} \cdot \sqrt[3]{a+1} \cdot \sqrt[5]{\frac{(1+a)^2}{(1-a)^3}} = \frac{(1-a)^{\frac{1}{2}}}{(1+a)^{\frac{1}{2}}} \cdot (a+1)^{\frac{1}{3}} \cdot \frac{(1+a)^{\frac{2}{5}}}{(1-a)^{\frac{3}{5}}} = \frac{(1+a)^{\frac{1}{3} + \frac{2}{5} - \frac{1}{2}}}{(1-a)^{\frac{3}{5} - \frac{1}{2}}} = \frac{(1+a)^{\frac{7}{30}}}{(1-a)^{\frac{1}{10}}} = \frac{\sqrt[30]{(1+a)^7}}{\sqrt[10]{1-a}}$
20. $\sqrt[3]{x+1} \cdot \sqrt{x^2-1} : \sqrt[6]{(x+1)^5} = (x+1)^{\frac{1}{3}} \cdot (x-1)^{\frac{1}{2}} \cdot (x+1)^{\frac{1}{2}} : (x+1)^{\frac{5}{6}} = (x-1)^{\frac{1}{2}} \cdot (x+1)^{\frac{1}{3} + \frac{1}{2} - \frac{5}{6}} = \sqrt{x-1}$
21. $(\sqrt[4]{x-y} : \sqrt[3]{x^2-y^2}) \cdot \sqrt[6]{3x+3y} = \frac{(x-y)^{\frac{1}{4}}}{(x-y)^{\frac{1}{3}} \cdot (x+y)^{\frac{1}{3}}} \cdot 3^{\frac{1}{6}} (x+y)^{\frac{1}{6}} =$
22. $= \frac{3^{\frac{1}{6}}}{(x-y)^{\frac{1}{3} - \frac{1}{4}} \cdot (x+y)^{\frac{1}{3} - \frac{1}{6}}} = \frac{3^{\frac{2}{12}}}{(x-y)^{\frac{1}{12}} \cdot (x+y)^{\frac{2}{12}}} = \sqrt[12]{\frac{9}{(x-y)(x+y)^2}}$
23. $\sqrt[3]{(a+b)^2} \cdot \sqrt[4]{\frac{4}{a+b}} : \sqrt[6]{8(a+b)^2} = (a+b)^{\frac{2}{3}} \cdot \frac{2^{\frac{2}{4}}}{(a+b)^{\frac{1}{4}}} \cdot \frac{1}{2^{\frac{3}{6}} (a+b)^{\frac{2}{6}}} = (a+b)^{\frac{1}{12}} = \sqrt[12]{a+b}$
24. $\sqrt[3]{2ab} \cdot \sqrt{a+b} \cdot \sqrt[6]{4ab^3} : \sqrt{\frac{2a+2b}{ab}} = 2^{\frac{1}{3}} a^{\frac{1}{3}} b^{\frac{1}{3}} (a+b)^{\frac{1}{2}} 2^{\frac{2}{6}} a^{\frac{1}{6}} b^{\frac{3}{6}} \cdot \frac{a^{\frac{1}{2}} b^{\frac{1}{2}}}{2^{\frac{1}{2}} (a+b)^{\frac{1}{2}}} =$
25. $= 2^{\frac{2}{3} - \frac{1}{2}} a b^{\frac{5}{6} + \frac{1}{2}} = 2^{\frac{1}{6}} a b^{\frac{4}{3}} = a \sqrt[6]{2 b^8}$
26. $\sqrt{3^2} \cdot \sqrt{6} = \sqrt{54}$ $\sqrt[3]{2^2} \cdot \sqrt[6]{2} = \sqrt[6]{32}$
27. $\sqrt[5]{a} \cdot \sqrt[3]{a^2} = \sqrt[15]{a^{13}}$ $\sqrt[5]{ab} \cdot \sqrt[4]{a^3} = \sqrt[20]{a^{19} b^4}$
28. $\sqrt[3]{a^2} \cdot \sqrt[4]{a} = \sqrt[12]{a^{11}}$ $\sqrt[12]{2^5} \cdot \sqrt[18]{2a^2} = \sqrt[36]{2^{17} a^4}$
29. $\sqrt[3]{a^2} \cdot \sqrt[4]{a^2 b^3} = \sqrt[12]{a^{14} b^9}$ $\sqrt[4]{ab^2} \cdot \sqrt[8]{a^3} = \sqrt[8]{a^5 b^4}$
30. $\sqrt[5]{ab^3} \cdot \sqrt[4]{a^2 b} = \sqrt[20]{a^{14} b^{13}}$ $\sqrt[3]{b^2 c} \cdot \sqrt[6]{bc} = \sqrt[6]{b^5 c^3}$
31. $\frac{\sqrt[5]{a-b} \cdot \sqrt[3]{(a-b)^2}}{\sqrt[5]{(a-b)^3}} = \sqrt[15]{(a-b)^4}$
32. $(\sqrt[10]{a^3 b^4} : \sqrt[15]{ab}) : \sqrt[6]{ab^2} = \sqrt[15]{a}$
33. $a + 4b + 4\sqrt{ab} = (\sqrt{a} + 2\sqrt{b})^2$ $a + 9 + 6\sqrt{a} = (\sqrt{a} + 3)^2$
34. $4a + 1 - 4\sqrt{a} = (1 - 2\sqrt{a})^2$ $9 + 4a + 12\sqrt{a} = (3 + 2\sqrt{a})^2$
35. $(\sqrt{a} + 2\sqrt{b})^2 = (\sqrt{a})^2 + 2(\sqrt{a})(2\sqrt{b}) + (2\sqrt{b})^2 = a + 4\sqrt{ab} + 4b$
36. $(\sqrt{x} - \sqrt{y})^2 = (\sqrt{x})^2 + 2(\sqrt{x})(-\sqrt{y}) + (-\sqrt{y})^2 = x - 2\sqrt{xy} + y$
37. $\sqrt{a}(\sqrt{a^3} + \sqrt{4a}) = a^{\frac{1}{2}}(a^{\frac{3}{2}} + 2^{\frac{2}{2}} a^{\frac{1}{2}}) = a^{\frac{1}{2} + \frac{3}{2}} + 2 a^{\frac{1}{2} + \frac{1}{2}} = a^2 + 2a$
38. $\sqrt{2x} \left(\sqrt{\frac{x}{2}} - \sqrt{\frac{2}{x}} \right) = 2^{\frac{1}{2}} x^{\frac{1}{2}} \frac{x^{\frac{1}{2}}}{2^{\frac{1}{2}}} - 2^{\frac{1}{2}} x^{\frac{1}{2}} \frac{2^{\frac{1}{2}}}{x^{\frac{1}{2}}} = x^{\frac{1}{2} + \frac{1}{2}} - 2^{\frac{1}{2} + \frac{1}{2}} = x - 2$
39. $\sqrt[5]{a-b} \cdot \sqrt[5]{(a-b)^4} + (\sqrt[6]{a+b})^6 - \sqrt[4]{(2a-3b)^4} =$
 $= (a-b)^{\frac{1}{5}} \cdot (a-b)^{\frac{4}{5}} + (a+b)^{\frac{6}{6}} - (2a-3b)^{\frac{4}{4}} = a-b + a+b - 2a+3b = 3b$

$$\begin{aligned}
 40. \quad & \sqrt{\frac{\sqrt{128}}{\sqrt{8}} + \left(\frac{\sqrt{60}}{\sqrt{6}}\right)^2} - \sqrt{50} \cdot \sqrt{2} + \left(\frac{\sqrt[3]{10}}{\sqrt[3]{2}}\right)^3 = \\
 & = \sqrt{\frac{2^{\frac{7}{2}}}{2^{\frac{3}{2}}} + \frac{60^{\frac{2}{2}}}{6^{\frac{2}{2}}} - 2^{\frac{1}{2}} 5^{\frac{2}{2}} 2^{\frac{1}{2}} + \frac{10^{\frac{3}{3}}}{2^{\frac{3}{3}}}} = \sqrt{2^{\frac{7}{2}-\frac{3}{2}} + \frac{60}{6} - 2^{\frac{1}{2}+\frac{1}{2}} 5 + \frac{10}{2}} = \sqrt{2^2 + 10 - 10 + 5} = \sqrt{9} = 3
 \end{aligned}$$

$$41. \quad \left(\sqrt{\frac{1}{a}} + 2\sqrt{a}\right)\sqrt{a} + (1 - \sqrt{2a})(1 + \sqrt{2a}) = 1 + 2a + 1 - 2a = 2$$

$$\begin{aligned}
 42. \quad & \left[\left(\sqrt[3]{\frac{x^2}{y}} : \sqrt[6]{\frac{x}{y^4}}\right) \cdot \sqrt[12]{x^{-5}y^{-4}}\right]^{12} = \left(\sqrt[3]{\frac{x^2}{y}} : \sqrt[6]{\frac{x}{y^4}}\right)^{12} \cdot (\sqrt[12]{x^{-5}y^{-4}})^{12} = \left(\sqrt[3]{\frac{x^2}{y}}\right)^{12} : \left(\sqrt[6]{\frac{x}{y^4}}\right)^{12} \cdot \frac{1}{x^5 y^4} = \\
 & = \left(\frac{x^2}{y}\right)^{\frac{12}{3}} : \left(\frac{x}{y^4}\right)^{\frac{12}{6}} \cdot \frac{1}{x^5 y^4} = \frac{x^8}{y^4} \cdot \frac{y^8}{x^2} \cdot \frac{1}{x^5 y^4} = x
 \end{aligned}$$

$$43. \quad \left(\sqrt[3]{\frac{x^2}{y}} : \sqrt[6]{\frac{x}{y^5}}\right)^2 = \left(\sqrt[3]{\frac{x^2}{y}}\right)^2 : \left(\sqrt[6]{\frac{x}{y^5}}\right)^2 = \left(\frac{x^2}{y}\right)^{\frac{2}{3}} \cdot \left(\frac{y^5}{x}\right)^{\frac{2}{6}} = \frac{x^{\frac{4}{3}}}{y^{\frac{2}{3}}} \cdot \frac{y^{\frac{5}{3}}}{x^{\frac{2}{3}}} = x y$$

$$44. \quad \sqrt[4]{\frac{4x^2y}{9ab^3}} : \sqrt[6]{\frac{8xy^2}{3a^2b}} = \frac{2^{\frac{2}{4}} x^{\frac{2}{4}} y^{\frac{1}{4}}}{3^{\frac{2}{4}} a^{\frac{1}{4}} b^{\frac{3}{4}}} \cdot \frac{3^{\frac{1}{6}} a^{\frac{2}{6}} b^{\frac{1}{6}}}{2^{\frac{3}{6}} x^{\frac{1}{6}} y^{\frac{2}{6}}} = \frac{x^{\frac{1}{2}-\frac{1}{6}} a^{\frac{1}{3}-\frac{1}{4}}}{3^{\frac{1}{2}-\frac{1}{6}} y^{\frac{1}{3}-\frac{1}{4}} b^{\frac{3}{4}-\frac{1}{6}}} = \frac{a^{\frac{1}{12}} x^{\frac{1}{4}}}{3^{\frac{1}{12}} y^{\frac{1}{12}} b^{\frac{7}{12}}} = \sqrt[12]{\frac{ax^4}{3^4 y b^7}}$$

$$\begin{aligned}
 45. \quad & \sqrt[4]{\frac{a^3 - b^3}{a^3 + b^3}} \cdot \sqrt[4]{\frac{a^2 - ab + b^2}{a^2 + ab + b^2}} \cdot \sqrt{a - b} = \\
 & = \left(\frac{(a - b)(a^2 + ab + b^2)}{(a + b)(a^2 - ab + b^2)} \cdot \frac{a^2 - ab + b^2}{a^2 + ab + b^2} \cdot \frac{(a + b)^2}{(a - b)^2}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{a + b}{a - b}}
 \end{aligned}$$

$$46. \quad \left(\frac{\sqrt[5]{a^2 + a^3} : \sqrt{a + 1}}{\sqrt[10]{a^5 + a^4}}\right)^5 = \left(\frac{a^4(1 + a)^2}{(a + 1)^5 a^4(a + 1)}\right)^{\frac{5}{10}} = \left(\frac{1}{(a + 1)^4}\right)^{\frac{1}{2}} = \frac{1}{(a + 1)^2}$$

$$47. \quad \sqrt[3]{\frac{a^2 + 2a + 1}{ab - b}} \cdot \sqrt[6]{\frac{a^2 - 2a + 1}{ab + b}} \cdot \sqrt[4]{\frac{b^2(a - 1)^2}{2a^2 + 4a + 2}} = \frac{(a + 1)^{\frac{2}{3}}}{b^{\frac{1}{3}}(a - 1)^{\frac{1}{3}}} \cdot \frac{(a - 1)^{\frac{2}{6}}}{b^{\frac{1}{6}}(a + 1)^{\frac{1}{6}}} \cdot \frac{b^{\frac{2}{4}}(a - 1)^{\frac{2}{4}}}{2^{\frac{1}{4}}(a + 1)^{\frac{2}{4}}} = \frac{\sqrt{a - 1}}{\sqrt[4]{2}}$$

$$\begin{aligned}
 48. \quad & \sqrt{\frac{4 - x^2}{x + 2}} \cdot \sqrt{\frac{x^3 + 8}{2 - x}} \cdot \sqrt{\frac{x^2}{x^2 + 2x}} = \\
 & = \frac{(2 - x)^{\frac{1}{2}}(2 + x)^{\frac{1}{2}}}{(x + 2)^{\frac{1}{2}}} \cdot \frac{(x + 2)^{\frac{1}{2}}(x^2 - 2x + 4)^{\frac{1}{2}}}{(2 - x)^{\frac{1}{2}}} \cdot \frac{x^{\frac{2}{2}}}{x^{\frac{1}{2}}(x + 2)^{\frac{1}{2}}} = \sqrt{x(x^2 - 2x + 4)}
 \end{aligned}$$

$$49. \sqrt[3]{\frac{x+2y}{x^2-4y^2}} \cdot \sqrt{\frac{x-2y}{x+2y}} \cdot \sqrt[4]{\frac{(x^3+2x^2y)^2}{x^4y^2}} =$$

$$= \frac{(x+2y)^{\frac{1}{3}}}{(x+2y)^{\frac{1}{3}}(x-2y)^{\frac{1}{3}}} \cdot \frac{(x-2y)^{\frac{1}{2}}}{(x+2y)^{\frac{1}{2}}} \cdot \frac{x^{\frac{4}{2}}(x+2y)^{\frac{2}{2}}}{x^{\frac{4}{2}}y^{\frac{2}{2}}} = \frac{(x-2y)^{\frac{1}{2}-\frac{1}{3}}}{y^{\frac{1}{2}}} = \frac{\sqrt[6]{x-2y}}{\sqrt{y}}$$

$$50. \sqrt[3]{\frac{(a-1)^4}{a^2+a}} : \left[\sqrt{\frac{a^2-1}{a}} \cdot \sqrt[6]{\left(\frac{a-1}{a+1}\right)^5} \right] = \frac{(a-1)^{\frac{4}{3}}}{a^{\frac{1}{3}}(a+1)^{\frac{1}{3}}} \cdot \frac{a^{\frac{1}{2}}}{(a-1)^{\frac{1}{2}}(a+1)^{\frac{1}{2}}} \cdot \frac{(a+1)^{\frac{5}{6}}}{(a-1)^{\frac{5}{6}}} = a^{\frac{1}{6}} = \sqrt[6]{a}$$

$$51. \sqrt[m]{\frac{(x^2+xy)^2}{ax^2}} \cdot \sqrt[m]{\frac{a^2+ab}{a^2-b^2}} \cdot \sqrt[m]{\frac{ab^2-b^3}{(x^2-y^2)^2}} =$$

$$= \left(\frac{x^2(x+y)^2}{ax^2} \cdot \frac{a(a+b)}{(a-b)(a+b)} \cdot \frac{b^2(a-b)}{(x-y)^2(x+y)^2} \right)^{\frac{1}{m}} = \left(\frac{b^2}{(x-y)^2} \right)^{\frac{1}{m}} = \sqrt[m]{\frac{b^2}{(x-y)^2}}$$

$$52. \sqrt[n]{\frac{x^3-2x^2y+xy^2}{(xy+y^2)(x-y)}} \cdot \sqrt[n]{\frac{x^3+y^3}{x^2-y^2}} \cdot \sqrt[n]{\frac{9xy^2+9y^3}{x^2-xy+y^2}} =$$

$$= \left(\frac{x(x-y)^2}{y(x+y)(x-y)} \cdot \frac{(x+y)(x^2-xy+y^2)}{(x-y)(x+y)} \cdot \frac{9y^2(x+y)}{x^2-xy+y^2} \right)^{\frac{1}{n}} = (9xy)^{\frac{1}{n}} = \sqrt[n]{9xy}$$

$$53. a^{n^2-1} \sqrt{\frac{a+b}{a}} : \left(a^{n-1} \sqrt{\frac{a+b}{a}} \cdot a^{n+1} \sqrt{\frac{a}{a+b}} \right) =$$

$$= \left(\frac{a+b}{a} \cdot \frac{a^{n+1}}{(a+b)^{n+1}} \cdot \frac{(a+b)^{n-1}}{a^{n-1}} \right)^{\frac{1}{n^2-1}} = \left(\frac{a^{n+1-1-(n-1)}}{(a+b)^{n+1-1-(n-1)}} \right)^{\frac{1}{n^2-1}} = a^{n^2-1} \sqrt{\frac{a}{a+b}}$$

$$54. \sqrt[3]{\frac{x+y}{a}} \cdot \sqrt[4]{\frac{(x-y)^2}{x+y}} : \left(\sqrt[6]{\frac{x+y}{a^3}} \cdot \sqrt[4]{\frac{(x-y)^2}{a}} \right) =$$

$$= \frac{(x+y)^{\frac{1}{3}}}{a^{\frac{1}{3}}} \cdot \frac{(x-y)^{\frac{2}{4}}}{(x+y)^{\frac{1}{4}}} \cdot \frac{a^{\frac{3}{6}}}{(x+y)^{\frac{1}{6}}} \cdot \frac{a^{\frac{1}{4}}}{(x-y)^{\frac{2}{4}}} = \frac{a^{\frac{1}{2}+\frac{1}{4}-\frac{1}{3}}}{(x+y)^{\frac{1}{4}+\frac{1}{6}-\frac{1}{3}}} = \frac{a^{\frac{5}{12}}}{(x+y)^{\frac{1}{12}}} = \sqrt[12]{\frac{a^5}{x+y}}$$