

POTENZA DI UN RADICALE IN \mathbb{R}_0^+

1. $(\sqrt{x+1})^2 = (x+1)^{\frac{1}{2} \cdot 2} = x+1$ $(\sqrt{x+y})^3 = (x+y)^{\frac{1}{2} \cdot 3} = (x+y) \sqrt{x+y}$
2. $(\sqrt{a^2 - b^2})^4 = (a^2 - b^2)^{\frac{1}{2} \cdot 4} = (a^2 - b^2)^2$ $(\sqrt{3x})^4 = (3x)^{\frac{1}{2} \cdot 4} = (3x)^2 = 9x^2$
3. $(b \sqrt{b-2})^2 = b^2 (\sqrt{b-2})^2 = b^2 (b-2)$ $(\sqrt{a-b})^4 = (a-b)^{\frac{1}{2} \cdot 4} = (a-b)^2$
4. $(\sqrt{x+a})^3 = (x+a)^{\frac{1}{2} \cdot 3} = (x+a) \sqrt{x+a}$ $(\sqrt[3]{a+b})^4 = (a+b)^{\frac{1}{3} \cdot 4} = (a+b) \sqrt[3]{a+b}$
5. $(\sqrt[3]{2+ab})^6 = (2+ab)^{\frac{1}{3} \cdot 6} = (2+ab)^2$ $(\sqrt{x-1})^4 = (x-1)^{\frac{1}{2} \cdot 4} = (x-1)^2$
6. $(\sqrt[6]{9ab^2})^4 = (3^{\frac{2}{6}} a^{\frac{1}{6}} b^{\frac{2}{6}})^4 = 3^{\frac{4}{3}} a^{\frac{2}{3}} b^{\frac{4}{3}} = 3b \sqrt[3]{3a^2b}$
7. $(\sqrt[15]{32ab^3})^5 = (2^{\frac{5}{15}} a^{\frac{1}{15}} b^{\frac{3}{15}})^5 = 2^{\frac{5}{3}} a^{\frac{1}{3}} b = 2b \sqrt[3]{2^2a}$
8. $(\sqrt[4]{2a^2b^3})^2 = (2^{\frac{1}{4}} a^{\frac{2}{4}} b^{\frac{3}{4}})^2 = 2^{\frac{1}{2}} ab^{\frac{3}{2}} = ab \sqrt{2b}$
9. $(\sqrt[4]{a^3b^5})^3 = (a^{\frac{3}{4}} b^{\frac{5}{4}})^3 = a^{\frac{9}{4}} b^{\frac{15}{4}} = a^2 b^3 \sqrt[4]{ab^3}$
10. $(\sqrt[4]{(x-1)^2(x+1)})^2 = \left[(x-1)^{\frac{2}{4}} (x+1)^{\frac{1}{4}} \right]^2 = (x-1)(x+1)^{\frac{1}{2}} = (x-1) \sqrt{x+1}$
11. $(\sqrt[10]{(x-y)^3})^5 = \left[(x-y)^{\frac{3}{10}} \right]^5 = (x-y)^{\frac{3}{2}} = (x-y) \sqrt{x-y}$
12. $(\sqrt[n]{2 \cdot 5^{n-1}})^{2n} = \left(2^{\frac{1}{n}} \cdot 5^{\frac{n-1}{n}} \right)^{2n} = 2^2 \cdot 5^{2n-2}$
13. $(\sqrt[n-1]{2^{n+1}a^2})^2 = \left(2^{\frac{n+1}{n-1}} a^{\frac{2}{n-1}} \right)^2 = 2^{\frac{2n+2}{n-1}} a^{\frac{4}{n-1}} = 2^{2+\frac{4}{n-1}} a^{\frac{4}{n-1}} = 2^{2n-1} \sqrt[2]{2^4 a^4}$
14. $(\sqrt[6n]{xy^2})^{3n^2} = \left(x^{\frac{1}{6n}} y^{\frac{2}{6n}} \right)^{3n^2} = x^{\frac{n}{2}} y^n = y^n \sqrt{x^n}$
15. $(\sqrt[4]{2^n a^{3+n} x^3})^{2n} = \left(2^{\frac{n}{4}} a^{\frac{3+n}{4}} x^{\frac{3}{4}} \right)^{2n} = 2^{\frac{n^2}{2}} a^{\frac{3n+n^2}{2}} x^{\frac{3n}{2}} = \sqrt{2^{n^2} a^{3n+n^2} x^{3n}}$
16. $(\sqrt[6m]{4^m a^{n+1} b})^3 = \left(2^{\frac{2m}{6m}} a^{\frac{n+1}{6m}} b^{\frac{1}{6m}} \right)^3 = 2 a^{\frac{n+1}{2m}} b^{\frac{1}{2m}} = 2 \sqrt[2m]{a^{n+1} b}$
17. $(\sqrt[2mn]{5x^2y})^{m^2} = (5x^2y)^{\frac{m^2}{2mn}} = (5x^2y)^{\frac{m}{2n}} = \sqrt[2n]{5^m x^{2m} y^m}$
18. $(2 + \sqrt{2})^2 = 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$
19. $(1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + 2\sqrt{2}$
20. $(\sqrt{2} - \sqrt{3})^2 = 2 - 2\sqrt{6} + 3 = 5 - 2\sqrt{6}$
21. $(2 + \sqrt{2})^3 = 8 + 12\sqrt{2} + 12 + 2\sqrt{2} = 20 + 14\sqrt{2}$

22. $(1 - \sqrt{3})^3 = 1 - 3\sqrt{3} + 9 - 3\sqrt{3} = 10 - 6\sqrt{3}$
23. $(\sqrt{2} + 2\sqrt{3})^3 = 2\sqrt{2} + 12\sqrt{3} + 36\sqrt{2} + 24\sqrt{3} = 38\sqrt{2} + 36\sqrt{3}$
24. $(a + \sqrt{a})^2 = a^2 + 2a\sqrt{a} + a$
25. $(2\sqrt{a} - a\sqrt{2})^2 = 4a - 4a\sqrt{2a} + 2a^2$
26. $(2\sqrt{2} + 3)^2 = 8 + 12\sqrt{2} + 9 = 17 + 12\sqrt{2}$
27. $(3\sqrt{2} + 2\sqrt{3})^2 = 18 + 12\sqrt{6} + 12 = 30 + 12\sqrt{6}$
28. $(2 - 3\sqrt{2})^3 = 8 - 36\sqrt{2} + 108 - 54\sqrt{2} = 116 - 90\sqrt{2}$
29. $(1 + \sqrt[3]{2})^3 = 1 + 3\sqrt[3]{2} + 3\sqrt[3]{4} + 2 = 3 + 3\sqrt[3]{2} + 3\sqrt[3]{4}$
30. $\left(3\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 = 27 - 2 \cdot 3\sqrt{3} \cdot \frac{1}{\sqrt{3}} + \frac{1}{3} = 27 - 6 + \frac{1}{3} = \frac{64}{3}$
31. $\left(\sqrt[6]{\frac{4}{5}} + \sqrt{\frac{5}{2}}\right)^2 = \left(\sqrt[6]{\frac{4}{5}}\right)^2 + 2\left(\sqrt[6]{\frac{4}{5}}\right)\left(\sqrt{\frac{5}{2}}\right) + \frac{5}{2} = \sqrt[3]{\frac{4}{5}} + 2\sqrt[6]{\frac{25}{2}} + \frac{5}{2}$
32. $\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)^2 = \frac{a}{b} - 2\sqrt{\frac{a}{b}}\sqrt{\frac{b}{a}} + \frac{b}{a} = \frac{a}{b} - 2 + \frac{b}{a} = \frac{a^2 - 2ab + b^2}{ab} = \frac{(a-b)^2}{ab}$
33. $\left(2\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = 8 - 2 \cdot 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} = 8 - 4 + \frac{1}{2} = \frac{9}{2}$
34. $(1 - \sqrt[6]{5})^2 = 1 - 2\sqrt[6]{5} + \sqrt[3]{5}$
35. $(\sqrt[3]{2} + 2\sqrt[3]{4})^2 = \sqrt[3]{4} + 4\sqrt[3]{2} \cdot \sqrt[3]{4} + 4\sqrt[3]{2^4} = \sqrt[3]{4} + 8 + 8\sqrt[3]{2}$

ESTRAZIONE DI RADICE DA UN RADICALE IN \mathbb{R}_0^+

$$36. \quad \sqrt[3]{\sqrt{5}} = \left(5^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{6}} = \sqrt[6]{5} \qquad \sqrt{\sqrt{3a}} = \left((3a)^{\frac{1}{2}}\right)^{\frac{1}{2}} = (3a)^{\frac{1}{4}} = \sqrt[4]{3a}$$

$$37. \quad \sqrt{\sqrt[3]{3a^2b}} = \left((3a^2b)^{\frac{1}{3}}\right)^{\frac{1}{2}} = \sqrt[6]{3a^2b} \qquad \sqrt[4]{\sqrt[3]{12}} = \left((12)^{\frac{1}{3}}\right)^{\frac{1}{4}} = \sqrt[12]{12}$$

$$38. \quad \sqrt[3]{\sqrt[3]{81}} = \left((81)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \sqrt[9]{81} \qquad \sqrt[3]{\sqrt[5]{8}} = \left((2^3)^{\frac{1}{5}}\right)^{\frac{1}{3}} = \sqrt[15]{2}$$

$$39. \quad \sqrt[3]{\sqrt[3]{2a^2}} = \left((2a^2)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \sqrt[9]{2a^2} \qquad \sqrt[3]{\sqrt[5]{16a^2}} = \left((16a^2)^{\frac{1}{5}}\right)^{\frac{1}{3}} = \sqrt[15]{16a^2}$$

$$40. \quad \sqrt[3]{4\sqrt{2}} = \sqrt[3]{2^2 \cdot 2^{\frac{1}{2}}} = \sqrt[3]{2^{\frac{5}{2}}} = \left(2^{\frac{5}{2}}\right)^{\frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5}$$

$$41. \quad \sqrt{3\sqrt[3]{9}} = \sqrt{3 \cdot 3^{\frac{2}{3}}} = \sqrt{3^{\frac{5}{3}}} = \left(3^{\frac{5}{3}}\right)^{\frac{1}{2}} = \sqrt[6]{3^5}$$

$$42. \quad \sqrt{2\sqrt[3]{2a^2}} = \sqrt{2 \cdot 2^{\frac{1}{3}} a^{\frac{2}{3}}} = \left(2^{\frac{4}{3}} a^{\frac{2}{3}}\right)^{\frac{1}{2}} = 2^{\frac{2}{3}} a^{\frac{1}{3}} = \sqrt[3]{4a}$$

$$43. \quad \sqrt[3]{a^2 \sqrt[5]{\frac{1}{a^4}}} = \sqrt[3]{a^2 \cdot \frac{1}{a^{\frac{4}{5}}}} = \sqrt[3]{a^{\frac{6}{5}}} = \left(a^{\frac{6}{5}}\right)^{\frac{1}{3}} = a^{\frac{2}{5}} = \sqrt[5]{a^2}$$

$$44. \quad \sqrt{x \sqrt{x \sqrt{x}}} = \sqrt{x \sqrt{x \cdot x^{\frac{1}{2}}}} = \sqrt{x \sqrt{x^{\frac{3}{2}}}} = \sqrt{x \cdot x^{\frac{3}{4}}} = \sqrt{x^{\frac{7}{4}}} = x^{\frac{7}{8}} = \sqrt[8]{x^7}$$