ESPRESSIONI NELL'INSIEME DEI NUMERI REALI POSITIVI

1. $\sqrt{\sqrt[3]{2}} \cdot \sqrt[4]{2} = \left(2^{\frac{1}{3}}\right)^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} = 2^{\frac{1}{6}} \cdot 2^{\frac{1}{4}} = 2^{\frac{1}{6}+\frac{1}{4}} = 2^{\frac{5}{12}} = \sqrt[12]{2^5}$

2. $\sqrt[4]{\sqrt[5]{6}} \cdot \sqrt{\sqrt[5]{4}} : \sqrt[10]{\sqrt{3}} = \left((2\cdot 3)^{\frac{1}{5}}\right)^{\frac{1}{4}} \cdot \left(2^{\frac{2}{5}}\right)^{\frac{1}{2}} : \left(3^{\frac{1}{2}}\right)^{\frac{1}{10}} = 2^{\frac{1}{20}} \cdot 3^{\frac{1}{20}} \cdot 2^{\frac{1}{5}} : 3^{\frac{1}{20}} = 2^{\frac{5}{20}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$

3. $\dfrac{\sqrt[4]{\sqrt[3]{5}}}{\sqrt[3]{\sqrt{5}}} = \left(5^{\frac{1}{3}}\right)^{\frac{1}{4}} : \left(5^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{12}} : 5^{\frac{1}{6}} = 5^{-\frac{1}{12}} = \dfrac{1}{\sqrt[12]{5}}$

4. $\dfrac{\sqrt{\sqrt[5]{8}}}{\sqrt[5]{\sqrt[3]{2}}} = \left(2^{\frac{3}{5}}\right)^{\frac{1}{2}} : \left(2^{\frac{1}{3}}\right)^{\frac{1}{5}} = 2^{\frac{3}{10}} : 2^{\frac{1}{15}} = 2^{\frac{7}{30}} = \sqrt[30]{2^7}$

5. $\sqrt[6]{5} \cdot \sqrt[4]{\sqrt{10}} \cdot \sqrt[3]{\sqrt{2}} = 5^{\frac{1}{6}} \cdot \left((2\cdot 5)^{\frac{1}{2}}\right)^{\frac{1}{4}} \cdot \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{6}} \cdot 2^{\frac{1}{8}} \cdot 5^{\frac{1}{8}} \cdot 2^{\frac{1}{6}} = 2^{\frac{7}{24}} \cdot 5^{\frac{7}{24}} = 10^{\frac{7}{24}} = \sqrt[24]{10^7}$

6. $\dfrac{\sqrt[5]{\sqrt[3]{3}} \cdot \sqrt[3]{\sqrt{2}}}{\sqrt[6]{\sqrt[5]{6}}} = \left(3^{\frac{1}{3}}\right)^{\frac{1}{5}} \cdot \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} : \left((2\cdot 3)^{\frac{1}{5}}\right)^{\frac{1}{6}} = 3^{\frac{1}{15}-\frac{1}{30}} \cdot 2^{\frac{1}{6}-\frac{1}{30}} = 3^{\frac{1}{30}} \cdot 2^{\frac{4}{30}} = \sqrt[30]{3\cdot 2^4} = \sqrt[30]{48}$

7. $\sqrt[5]{x^2\,\sqrt[7]{x}} \cdot \sqrt[7]{x^3\,\sqrt{x}} : (\sqrt[4]{x})^2 = \left(x^2 \cdot x^{\frac{1}{7}}\right)^{\frac{1}{5}} \cdot \left(x^3 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{7}} : \left(x^{\frac{1}{4}}\right)^2 = x^{\frac{3}{7}} \cdot x^{\frac{1}{2}} : x^{\frac{1}{2}} = x^{\frac{3}{7}} = \sqrt[7]{x^3}$

8. $\sqrt{\dfrac{1}{2}\sqrt{2}} \cdot \sqrt{\dfrac{1}{2}\sqrt[3]{4}} \cdot \sqrt[3]{\sqrt{2+\dfrac{1}{2}}} = \left(\dfrac{1}{2}\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left(\dfrac{1}{2}\cdot 2^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot \left(\left(\dfrac{5}{2}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \dfrac{1}{2^{\frac{1}{4}}} \cdot \dfrac{1}{2^{\frac{1}{6}}} \cdot \dfrac{5^{\frac{1}{6}}}{2^{\frac{1}{6}}} = \dfrac{5^{\frac{1}{6}}}{2^{\frac{7}{12}}} = \sqrt[12]{\dfrac{5^2}{2^7}}$

9. $\sqrt{a\sqrt{a}} \cdot \sqrt[3]{a\sqrt{a}} = \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{3}} = a^{\frac{3}{4}} \cdot a^{\frac{1}{2}} = a^{\frac{5}{4}} = a\sqrt[4]{a}$

10. $\sqrt[4]{3x^2 y\sqrt{2xy\sqrt[3]{2xy^3}}} : \sqrt[12]{108} = \sqrt[4]{3x^2 y\sqrt{2xy\cdot 2^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{3}{3}}}} : \sqrt[12]{2^2\cdot 3^3} =$

$= \sqrt[4]{3x^2 y\cdot 2^{\frac{2}{3}}x^{\frac{2}{3}}y} : \sqrt[12]{2^2\cdot 3^3} = 3^{\frac{1}{4}}\cdot 2^{\frac{1}{6}}\, x^{\frac{2}{3}} y^{\frac{1}{2}} : \left(2^{\frac{1}{6}}\cdot 3^{\frac{1}{4}}\right) = x^{\frac{4}{6}} y^{\frac{3}{6}} = \sqrt[6]{x^4 y^3}$

11. $\sqrt[3]{a^2 b\sqrt{5a^2}} : \left(\sqrt[3]{a\sqrt{a+b}} \cdot \sqrt{(a+b)\sqrt[3]{a\sqrt{5}}}\right)^2 =$

$= \sqrt[3]{a^2 b\cdot 5^{\frac{1}{2}}a^{\frac{2}{2}}} : \left(\sqrt[3]{a(a+b)^{\frac{1}{2}}} \cdot \sqrt{(a+b)a^{\frac{1}{3}}5^{\frac{1}{6}}}\right)^2 = = 5^{\frac{1}{6}}\, a^{\frac{3}{3}}\, b^{\frac{1}{3}} : \left(a^{\frac{1}{3}}(a+b)^{\frac{1}{6}}\,(a+b)^{\frac{1}{2}}\,a^{\frac{1}{6}}\,5^{\frac{1}{12}}\right)^2 =$

$= 5^{\frac{1}{6}}\, a\, b^{\frac{1}{3}} : \left((a+b)^{\frac{2}{3}} a^{\frac{1}{2}} 5^{\frac{1}{12}}\right)^2 = \dfrac{5^{\frac{1}{6}}\, a\, b^{\frac{1}{3}}}{(a+b)^{\frac{4}{3}}\, a\, 5^{\frac{1}{6}}} = \dfrac{1}{a+b}\,\sqrt[3]{\dfrac{b}{a+b}}$

12. $\sqrt{a\sqrt[3]{a\sqrt[3]{a^2}}}\cdot\sqrt[3]{a\sqrt[3]{\frac{1}{a}}}:\sqrt{\frac{1}{a}}=\sqrt{a\sqrt[3]{a\cdot a^{\frac{2}{3}}}}\cdot\sqrt[3]{a\,\frac{1}{a^{\frac{1}{3}}}\cdot a^{\frac{1}{2}}}=\sqrt{a\cdot a^{\frac{5}{9}}}\cdot\sqrt[3]{a^{\frac{7}{6}}}=$

$=\sqrt{a^{\frac{14}{9}}}\cdot a^{\frac{7}{18}}=a^{\frac{7}{9}}\cdot a^{\frac{7}{18}}=a^{\frac{21}{18}}=a^{\frac{7}{6}}=a^{1+\frac{1}{6}}=a\sqrt[6]{a}$

13. $\sqrt[5]{x\sqrt[7]{x^3}}\cdot\sqrt{x\sqrt[7]{\frac{1}{x^2}}}:\sqrt[7]{x^4\sqrt{x}}=\sqrt[5]{x\cdot x^{\frac{3}{7}}}\cdot\sqrt{x\cdot\frac{1}{x^{\frac{2}{7}}}}:\sqrt[7]{x^4\cdot x^{\frac{1}{2}}}=\sqrt[5]{x^{\frac{10}{7}}}\cdot\sqrt{x^{\frac{5}{7}}}:\sqrt[7]{x^{\frac{9}{2}}}=$

$=x^{\frac{2}{7}}\cdot x^{\frac{5}{14}}:x^{\frac{9}{14}}=x^{\frac{2}{7}+\frac{5}{14}-\frac{9}{14}}=x^0=1$

14. $\sqrt[3]{\frac{a-b}{b^3}\sqrt{\frac{a+b}{ab}}}:\left(\sqrt[3]{\frac{a+b}{a^3}\sqrt{\frac{a-b}{ab}}}:\sqrt[6]{\frac{a+b}{a-b}}\right)=\sqrt[3]{\frac{a-b}{b^3}\cdot\frac{(a+b)^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}}:\left(\sqrt[3]{\frac{a+b}{a^3}\cdot\frac{(a-b)^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}}:\sqrt[6]{\frac{a+b}{a-b}}\right)=$

$=\frac{(a-b)^{\frac{1}{3}}(a+b)^{\frac{1}{6}}}{a^{\frac{1}{6}}b^{\frac{7}{6}}}:\left(\frac{(a+b)^{\frac{1}{3}}(a-b)^{\frac{1}{6}}}{a^{\frac{7}{6}}b^{\frac{1}{6}}}\cdot\frac{(a-b)^{\frac{1}{6}}}{(a+b)^{\frac{1}{6}}}\right)=$

$=\frac{(a-b)^{\frac{1}{3}}(a+b)^{\frac{1}{6}}}{a^{\frac{1}{6}}b^{\frac{7}{6}}}\cdot\frac{a^{\frac{7}{6}}b^{\frac{1}{6}}}{(a+b)^{\frac{1}{6}}(a-b)^{\frac{1}{3}}}=\frac{a}{b}$

15. $\sqrt{2x\sqrt[3]{2x}}\cdot\sqrt{\frac{1}{2x}\sqrt[3]{\frac{1}{2x}}}:\sqrt[4]{x\sqrt[3]{2x}}=\sqrt{2x\cdot 2^{\frac{1}{3}}x^{\frac{1}{3}}}\cdot\sqrt{\frac{1}{2x}\cdot\frac{1}{2^{\frac{1}{3}}x^{\frac{1}{3}}}}:\sqrt[4]{x\cdot 2^{\frac{1}{3}}x^{\frac{1}{3}}}=$

$=\sqrt{2x\cdot 2^{\frac{1}{3}}x^{\frac{1}{3}}\cdot\frac{1}{2^{\frac{1}{2}}x^{\frac{1}{2}}}\cdot\frac{1}{2^{\frac{1}{6}}x^{\frac{1}{6}}}}:\sqrt[4]{2^{\frac{1}{3}}x^{\frac{4}{3}}}=\sqrt{2^{\frac{2}{3}}x^{\frac{2}{3}}}:\left(2^{\frac{1}{12}}x^{\frac{1}{3}}\right)=\frac{2^{\frac{1}{3}}x^{\frac{1}{3}}}{2^{\frac{1}{12}}x^{\frac{1}{3}}}=2^{\frac{1}{4}}=\sqrt[4]{2}$

16. $\sqrt{2a\sqrt[5]{2a}}\cdot\sqrt[3]{2b\sqrt{2b}}:\left(\sqrt[5]{\sqrt{2^3a^2b}}\cdot\sqrt[3]{2ab\sqrt[5]{ab}}\right)=$

$=\sqrt{2a\cdot 2^{\frac{1}{5}}a^{\frac{1}{5}}}\cdot\sqrt[3]{2b\cdot 2^{\frac{1}{2}}b^{\frac{1}{2}}}:\left(\sqrt[5]{2^{\frac{3}{2}}ab^{\frac{1}{2}}}\cdot\sqrt[3]{2ab\,a^{\frac{1}{5}}b^{\frac{1}{5}}}\right)=$

$=\sqrt{2^{\frac{6}{5}}a^{\frac{6}{5}}}\cdot\sqrt[3]{2^{\frac{3}{2}}b^{\frac{3}{2}}}:\left(2^{\frac{3}{10}}a^{\frac{1}{5}}b^{\frac{1}{10}}\cdot 2^{\frac{1}{3}}a^{\frac{2}{5}}b^{\frac{2}{5}}\right)==2^{\frac{3}{5}}a^{\frac{3}{5}}\cdot 2^{\frac{1}{2}}b^{\frac{1}{2}}:\left(2^{\frac{19}{30}}a^{\frac{3}{5}}b^{\frac{1}{2}}\right)=2^{\frac{3}{5}+\frac{1}{2}-\frac{19}{30}}=2^{\frac{7}{15}}=\sqrt[15]{2^7}$

17. $\sqrt[3]{(a+b)^2\sqrt{(a+b)^3\sqrt{a+b}}}\cdot\sqrt{(a+b)\sqrt{a+b}}=$

$=\sqrt[3]{(a+b)^2\sqrt{(a+b)^3(a+b)^{\frac{1}{2}}}}\cdot\sqrt{(a+b)(a+b)^{\frac{1}{2}}}=$

$=\sqrt[3]{(a+b)^2(a+b)^{\frac{7}{4}}}\cdot\sqrt{(a+b)^{\frac{3}{2}}}=\sqrt[3]{(a+b)^{\frac{15}{4}}}\cdot(a+b)^{\frac{3}{4}}=(a+b)^{\frac{5}{4}}\cdot(a+b)^{\frac{3}{4}}=(a+b)^2$

18. $\left(\sqrt[3]{2\sqrt[3]{2\sqrt{\frac{1}{2}}}}\cdot\sqrt[3]{\frac{1}{2}\sqrt[3]{2\sqrt[3]{\frac{1}{4}}}}\right)^6\cdot\sqrt[3]{2^2\sqrt{2}}=\left(\sqrt[3]{2\sqrt[3]{2\frac{1}{2^{\frac{1}{2}}}}}\cdot\sqrt[3]{\frac{1}{2}\sqrt[3]{2\frac{1}{2^{\frac{2}{3}}}}}\right)^6\cdot\sqrt[3]{2^2\cdot 2^{\frac{1}{2}}}=$

$=\left(\sqrt[3]{2\cdot 2^{\frac{1}{6}}}\cdot\sqrt[3]{\frac{1}{2}\cdot 2^{\frac{1}{9}}}\right)^6\cdot\sqrt[3]{2^{\frac{5}{2}}}=\left(2^{\frac{7}{18}}\cdot\sqrt[3]{\frac{1}{2^{\frac{8}{9}}}}\right)^6\cdot 2^{\frac{5}{6}}=\left(2^{\frac{7}{18}}\cdot\frac{1}{2^{\frac{8}{27}}}\right)^6\cdot 2^{\frac{5}{6}}=$

$=\left(2^{\frac{5}{54}}\right)^6\cdot 2^{\frac{5}{6}}=2^{\frac{5}{9}}\cdot 2^{\frac{5}{6}}=2^{\frac{25}{18}}=2^{1+\frac{7}{18}}=2\sqrt[18]{2^7}$

19. $\sqrt[3]{a\sqrt{a\sqrt[4]{a^3}}}\cdot\sqrt[3]{a\sqrt{\frac{1}{a}\sqrt{a}}}:\sqrt{a}=\sqrt[3]{a\sqrt{a\cdot a^{\frac{3}{4}}}}\cdot\sqrt[3]{a\sqrt{\frac{1}{a}a^{\frac{1}{2}}}}:a^{\frac{1}{2}}=\sqrt[3]{a\cdot a^{\frac{7}{8}}}\cdot\sqrt[3]{a\cdot\frac{1}{a^{\frac{1}{4}}}}:a^{\frac{1}{2}}=$

$=\sqrt[3]{a^{\frac{15}{8}}}\cdot\sqrt[3]{a^{\frac{3}{4}}}:a^{\frac{1}{2}}=a^{\frac{5}{8}}\cdot a^{\frac{1}{4}}:a^{\frac{1}{2}}=a^{\frac{5}{8}+\frac{1}{4}-\frac{1}{2}}=a^{\frac{3}{8}}=\sqrt[8]{a^3}$

20. $\left[\left(\sqrt{a\sqrt[3]{\frac{1}{a}}}\right)^3:\left(\sqrt[6]{a^3\sqrt{a}}\right)^3+2a\sqrt[4]{a^{-7}}\right]^2\cdot\frac{a\sqrt{a}}{9}=$

$=\left[\left(\sqrt{a\cdot\frac{1}{a^{\frac{1}{3}}}}\right)^3:\left(\sqrt[6]{a^3\cdot a^{\frac{1}{2}}}\right)^3+2a\cdot\frac{1}{a^{\frac{7}{4}}}\right]^2\cdot\frac{a\cdot a^{\frac{1}{2}}}{9}=\left[\left(\sqrt{a^{\frac{2}{3}}}\right)^3:\left(\sqrt[6]{a^{\frac{7}{2}}}\right)^3+\frac{2}{a^{\frac{3}{4}}}\right]^2\cdot\frac{a^{\frac{3}{2}}}{9}=$

$=\left[\left(a^{\frac{1}{3}}\right)^3:\left(a^{\frac{7}{12}}\right)^3+\frac{2}{a^{\frac{3}{4}}}\right]^2\cdot\frac{a^{\frac{3}{2}}}{9}=\left[a:a^{\frac{7}{4}}+\frac{2}{a^{\frac{3}{4}}}\right]^2\cdot\frac{a^{\frac{3}{2}}}{9}=\left[\frac{1}{a^{\frac{3}{4}}}+\frac{2}{a^{\frac{3}{4}}}\right]^2\cdot\frac{a^{\frac{3}{2}}}{9}=$

$=\left(\frac{3}{a^{\frac{3}{4}}}\right)^2\cdot\frac{a^{\frac{3}{2}}}{9}=\frac{9}{a^{\frac{3}{2}}}\cdot\frac{a^{\frac{3}{2}}}{9}=1$

21. $\sqrt[3]{(a^2+b)\sqrt{a^2+b}}\cdot\sqrt[3]{a^2\sqrt[5]{a^4}}:\sqrt{a^2+b}=\sqrt[3]{(a^2+b)^{\frac{3}{2}}}\cdot\sqrt[3]{a^2\cdot a^{\frac{4}{5}}}:(a^2+b)^{\frac{1}{2}}=$

$=\frac{(a^2+b)^{\frac{1}{2}}\cdot\sqrt[3]{a^{\frac{14}{5}}}}{(a^2+b)^{\frac{1}{2}}}=\sqrt[3]{a^{\frac{14}{5}}}=a^{\frac{14}{15}}=\sqrt[15]{a^{14}}$