# ESERCIZI DI RIEPILOGO SULL'INSIEME DEI NUMERI REALI POSITIVI

EQUAZIONI CON COEFFICIENTI IRRAZIONALI

1. $\sqrt{2}\,x = \sqrt{8}$ $\qquad x = \dfrac{\sqrt{8}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{\sqrt{2}} = 2$

2. $\sqrt{2}\,x = \sqrt{6}$ $\qquad x = \dfrac{\sqrt{6}}{\sqrt{2}} = \dfrac{2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{2^{\frac{1}{2}}} = \sqrt{3}$

3. $x + \sqrt{3} = 2\,(x + \sqrt{3})$ $\qquad x - 2x = -\sqrt{3} + 2\sqrt{3}$ $\qquad\qquad x = -\sqrt{3}$

4. $\sqrt{8}\,x + \sqrt{18}\,x = 5$ $\qquad 2\sqrt{2}\,x + 3\sqrt{2}\,x = 5$ $\qquad\qquad x = \dfrac{1}{\sqrt{2}}$

5. $\sqrt{3}\,x = \sqrt{6} + \sqrt{27}$ $\qquad \sqrt{3}\,x = \sqrt{6} + 3\sqrt{3}$ $\qquad\qquad x = \sqrt{2} + 3$

6. $2x + 2\sqrt{3} = \sqrt{3}\,x + 4$ $\qquad x\,(2 - \sqrt{3}) = 4 - 2\sqrt{3}$ $\qquad x = 2$

7. $\sqrt{3}\,(x - \sqrt{6} + 1) = \sqrt{3}\,(1 - x) + \sqrt{18}$ $\qquad \sqrt{3}\,x - 3\sqrt{2} + \sqrt{3} = \sqrt{3} - \sqrt{3}\,x + 3\sqrt{2}$

   $2\sqrt{3}\,x = 6\sqrt{2}$ $\qquad x = \dfrac{6\sqrt{2}}{2\sqrt{3}} = \dfrac{3\sqrt{2}}{\sqrt{3}} = \dfrac{3 \cdot 2^{\frac{1}{2}}}{3^{\frac{1}{2}}} = 3^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$

8. $x\,(\sqrt{5} - 5) + \sqrt{2}\,(x + 1) - \sqrt{2} = \sqrt{125} + x\,(\sqrt{2} - 5)$

   $x\sqrt{5} - 5x + x\sqrt{2} + \sqrt{2} - \sqrt{2} = 5\sqrt{5} + x\sqrt{2} - 5x$ $\qquad x = 5$

9. $x\sqrt{3} = x + 2\sqrt{3}$ $\qquad x\,(\sqrt{3} - 1) = 2\sqrt{3}$ $\qquad x = \dfrac{2\sqrt{3}}{\sqrt{3} - 1}$

   $x = \dfrac{2\sqrt{3}}{\sqrt{3} - 1} \cdot \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1} = \dfrac{2\,(3 + \sqrt{3})}{3 - 1} = 3 + \sqrt{3}$

10. $\sqrt{2}\,x = 2 + \sqrt{2}$ $\qquad x = \dfrac{2 + \sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}\,(\sqrt{2} + 1)}{\sqrt{2}} = \sqrt{2} + 1$

11. $2\sqrt{2}\,x + 3 + \sqrt{2} = \sqrt{3}\,x + \sqrt{2}\,(2\sqrt{3} + 1)$ $\qquad 2\sqrt{2}\,x + 3 + \sqrt{2} = \sqrt{3}\,x + 2\sqrt{6} + \sqrt{2}$

    $x\,(2\sqrt{2} - \sqrt{3}) = 2\sqrt{6} - 3$ $\qquad x = \dfrac{\sqrt{3}\,(2\sqrt{2} - \sqrt{3})}{(2\sqrt{2} - \sqrt{3})} = \sqrt{3}$

12. $(2 - \sqrt{3})\,x - \sqrt{6} = 2x - \sqrt{3}\,(2\sqrt{2} + 1)$ $\qquad 2x - \sqrt{3}\,x - \sqrt{6} = 2x - 2\sqrt{6} - \sqrt{3}$

    $-\sqrt{3}\,x = -\sqrt{3}\,(\sqrt{2} + 1)$ $\qquad x = \sqrt{2} + 1$

13. $x\,(3 - \sqrt{3}) = 6\left(\dfrac{x}{3 - \sqrt{3}} - 2\right)$ $\qquad$ $\dfrac{x\,(9 + 3 - 6\sqrt{3})}{3 - \sqrt{3}} = \dfrac{6x - 36 + 12\sqrt{3}}{3 - \sqrt{3}}$

$6x\,(1 - \sqrt{3}) = 12(-3 + \sqrt{3})$ $\qquad$ $x = \dfrac{12\sqrt{3}\,(-\sqrt{3} + 1)}{6\,(1 - \sqrt{3})} = 2\sqrt{3}$

14. $\dfrac{x - 2\sqrt{2} - 4}{6\sqrt{2}} + \dfrac{x + 2\sqrt{2}}{2\sqrt{2}} = \dfrac{x + 1}{3}$ $\qquad$ $\dfrac{x - 2\sqrt{2} - 4 + 3x + 6\sqrt{2}}{6\sqrt{2}} = \dfrac{2\sqrt{2}\,x + 2\sqrt{2}}{6\sqrt{2}}$

$4x - 2\sqrt{2}\,x = 4 - 2\sqrt{2}$ $\qquad$ $x = 1$

15. $\dfrac{x - 2}{\sqrt{3} - 1} + \dfrac{2x}{\sqrt{3} + 1} = 3$ $\qquad$ $\dfrac{\sqrt{3}\,x + x - 2\sqrt{3} - 2 + 2x\sqrt{3} - 2x - 6}{(\sqrt{3} - 1)\,(\sqrt{3} + 1)} = 0$ $\qquad$ $3x\sqrt{3} - x = 8 + 2\sqrt{3}$

$x = \dfrac{8 + 2\sqrt{3}}{3\sqrt{3} - 1} \cdot \dfrac{3\sqrt{3} + 1}{3\sqrt{3} + 1} = \dfrac{24\sqrt{3} + 8 + 18 + 2\sqrt{3}}{27 - 1} = \dfrac{26\,(\sqrt{3} + 1)}{26} = \sqrt{3} + 1$

16. $\dfrac{3\,(x - 3\sqrt{2})}{4\sqrt{3}} - \dfrac{x - 2\sqrt{3}}{3\sqrt{2}} = \dfrac{1}{2}$ $\qquad$ $\dfrac{9\sqrt{2}\,x - 54 - 4\sqrt{3}\,x + 24 - 6\sqrt{6}}{12\sqrt{6}} = 0$

$x\,(9\sqrt{2} - 4\sqrt{3}) = 30 + 6\sqrt{6}$

$x = \dfrac{30 + 6\sqrt{6}}{9\sqrt{2} - 4\sqrt{3}} \cdot \dfrac{9\sqrt{2} + 4\sqrt{3}}{9\sqrt{2} + 4\sqrt{3}} = \dfrac{6\,(45\sqrt{2} + 20\sqrt{3} + 18\sqrt{3} + 12\sqrt{2})}{162 - 48} =$

$= \dfrac{57\sqrt{2} + 38\sqrt{3}}{19} = \dfrac{19\,(3\sqrt{2} + 2\sqrt{3})}{19} = 3\sqrt{2} + 2\sqrt{3}$

17. $\dfrac{6}{7} + \dfrac{x}{3 - \sqrt{2}} = 2 - \dfrac{x}{7}\,(\sqrt{2} - 3)$ $\qquad$ $\dfrac{18 - 6\sqrt{2} + 7x - 42 + 14\sqrt{2} + 6\sqrt{2}\,x - 11x}{7\,(3 - \sqrt{2})} = 0$

$6\sqrt{2}\,x - 4x = -8\sqrt{2} + 24$ $\qquad$ $x = \dfrac{8\,(3 - \sqrt{2})}{2\sqrt{2}\,(3 - \sqrt{2})} = \dfrac{4}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$

18. $1 - \dfrac{x - 2 + \sqrt{2}}{2 - \sqrt{2}} = \dfrac{2 + \sqrt{2} - x}{2 + \sqrt{2}}$

$\dfrac{2 - 2x - x\sqrt{2} + 4 + 2\sqrt{2} - 2\sqrt{2} - 2}{(2 - \sqrt{2})\,(2 + \sqrt{2})} = \dfrac{4 - 2\sqrt{2} + 2\sqrt{2} - 2 - 2x + x\sqrt{2}}{(2 - \sqrt{2})\,(2 + \sqrt{2})}$

$-2x\sqrt{2} = -2$ $\qquad$ $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

19. $\dfrac{1}{x} + \dfrac{1}{x - \sqrt{2}} = \dfrac{\sqrt{2}}{x}$ $\qquad\qquad$ $\dfrac{x - \sqrt{2} + x - x\sqrt{2} + 2}{x(x - \sqrt{2})} = 0$ $\qquad\qquad$ $x \neq 0; \sqrt{2}$

$\qquad$ $2x - x\sqrt{2} = \sqrt{2} - 2$ $\qquad\qquad$ $x = -1$

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20. $\sqrt{2x^4} + \sqrt{8x^2 y^2} + \sqrt{2y^4} = x^2\sqrt{2} + 2xy\sqrt{2} + y^2\sqrt{2} = (x^2 + 2xy + y^2)\sqrt{2} = (x+y)^2\sqrt{2}$

21. $\sqrt{3a^2} \cdot \sqrt[4]{3a} + \sqrt[4]{27ab^2} \cdot \sqrt{b} = 3^{\frac{1}{2}} a \, 3^{\frac{1}{4}} a^{\frac{1}{4}} + 3^{\frac{3}{4}} a^{\frac{1}{4}} b^{\frac{2}{4}} b^{\frac{1}{2}} = 3^{\frac{3}{4}} a^{\frac{1}{4}} (a+b) = (a+b)\sqrt[4]{27a}$

22. $\sqrt{a^5} + \sqrt{4ab^2} - \sqrt{4a^3} - \sqrt{a^3 b^2} = a^2\sqrt{a} + 2b\sqrt{a} - 2a\sqrt{a} - ab\sqrt{a} = (a-2)(a-b)\sqrt{a}$

23. $\sqrt[3]{x^4 - x^3 y} + \sqrt[3]{xy^3 - y^4} - \sqrt[3]{x-y} = \sqrt[3]{x-y}\,(x+y-1)$

24. $\dfrac{3\sqrt{8} + \sqrt{125} - 2\sqrt{18} + 4\sqrt{50} - \sqrt{45}}{\sqrt{5} + 10\sqrt{2}} = \dfrac{6\sqrt{2} + 5\sqrt{5} - 6\sqrt{2} + 20\sqrt{2} - 3\sqrt{5}}{\sqrt{5} + 10\sqrt{2}} =$

$= \dfrac{2\sqrt{5} + 20\sqrt{2}}{\sqrt{5} + 10\sqrt{2}} = \dfrac{2(\sqrt{5} + 10\sqrt{2})}{\sqrt{5} + 10\sqrt{2}} = 2$

25. $\left[ \dfrac{\sqrt{3}}{\sqrt{12}} - 2\sqrt{3} + \dfrac{2}{3}\sqrt{\dfrac{3}{8}} : \dfrac{\sqrt{3}}{\sqrt{2}} - \sqrt{\left(\dfrac{1}{\sqrt[4]{3}}\right)^8} + \dfrac{\sqrt{150}}{\sqrt{2}} \right]^2 - \dfrac{109}{4} =$

$= \left[ \dfrac{\sqrt{3}}{2\sqrt{3}} - 2\sqrt{3} + \dfrac{2}{3}\dfrac{\sqrt{3}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{3}} - \sqrt{\dfrac{1}{9}} + \dfrac{5\sqrt{6}}{\sqrt{2}} \right]^2 - \dfrac{109}{4} =$

$= \left[ \dfrac{1}{2} - 2\sqrt{3} + \dfrac{1}{3} - \dfrac{1}{3} + 5\sqrt{3} \right]^2 - \dfrac{109}{4} = 27 + \dfrac{1}{4} + 3\sqrt{3} - \dfrac{109}{4} = 3\sqrt{3}$

26. $[(2\sqrt{5} - 3\sqrt{10})^2 - (2\sqrt{2} + 1)^2 + (\sqrt{3} - \sqrt{6})^2](11 + 7\sqrt{2}) =$

$= [20 + 90 - 60\sqrt{2} - 8 - 1 - 4\sqrt{2} + 3 + 6 - 6\sqrt{2}](11 + 7\sqrt{2}) =$

$= [110 - 70\sqrt{2}](11 + 7\sqrt{2}) = 10(11 - 7\sqrt{2})(11 + 7\sqrt{2}) = 10(121 - 98) = 230$

27. $\left(\sqrt{2\sqrt{5}}\right)^2 + (2 + \sqrt{5})^2 + (3 - \sqrt{5})^2 + (\sqrt{7} - 7\sqrt{2})(\sqrt{7} + 7\sqrt{2}) =$

$= 2\sqrt{5} + 4 + 5 + 4\sqrt{5} + 9 + 5 - 6\sqrt{5} + 7 - 98 = -68$

28. $\dfrac{1}{\sqrt{5} + \sqrt{3}} + \dfrac{1}{\sqrt{5} - \sqrt{3}} - \dfrac{3}{2}\sqrt{\dfrac{20}{9}} = \dfrac{\sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3}}{2} - \dfrac{3}{2} \cdot \dfrac{2\sqrt{5}}{3} = \sqrt{5} - \sqrt{5} = 0$

29.  $(\sqrt{2} + \sqrt[3]{5})^2 - (2\sqrt{5} - \sqrt[3]{2})^2 + (\sqrt[3]{5} + \sqrt[3]{2})^2 - 2(\sqrt[6]{200} + 2\sqrt[6]{500}) =$

$= 2 + \sqrt[3]{25} + 2\sqrt[6]{200} - 20 - \sqrt[3]{4} + 4\sqrt[6]{500} + \sqrt[3]{25} + \sqrt[3]{4} + 2\sqrt[3]{10} - 2\sqrt[6]{200} - 4\sqrt[6]{500} =$

$= 2\sqrt[3]{25} - 18 + 2\sqrt[3]{10}$

30.  $(\sqrt{a+1} - \sqrt{a-2})(\sqrt{a+1} + \sqrt{a-2}) = a + 1 - a + 2 = 3$

31.  $\sqrt[4]{\dfrac{a^3 + 3a^2 + 3a + 1}{a^2 - 2a + 1}} : \sqrt[3]{\dfrac{a+1}{(a-1)^2}} : \sqrt[6]{\dfrac{a^2+a}{a-1}} =$

$= \dfrac{(a+1)^{\frac{3}{4}}}{(a-1)^{\frac{2}{4}}} \cdot \dfrac{(a-1)^{\frac{2}{3}}}{(a+1)^{\frac{1}{3}}} \cdot \dfrac{(a-1)^{\frac{1}{6}}}{a^{\frac{1}{6}}(a+1)^{\frac{1}{6}}} = \dfrac{(a+1)^{\frac{1}{4}}(a-1)^{\frac{1}{3}}}{a^{\frac{1}{6}}} = {}^{12}\!\sqrt{\dfrac{(a+1)^3(a-1)^4}{a^2}}$

32.  $\dfrac{\sqrt{b}}{\sqrt{a}+\sqrt{b}} - \dfrac{\sqrt{b}}{\sqrt{a}-\sqrt{b}} + \dfrac{2a}{a-b} = \dfrac{\sqrt{ab} - b - \sqrt{ab} - b + 2a}{a-b} = \dfrac{2(a-b)}{a-b} = 2$

33.  $\left(\sqrt[3]{a\sqrt[5]{a^2\sqrt{a}}}\right)^2 = \left(\sqrt[3]{a\sqrt[5]{a^2\,a^{\frac{1}{2}}}}\right)^2 = \left(\sqrt[3]{a\sqrt[5]{a^{\frac{5}{2}}}}\right)^2 = \left(\sqrt[3]{a\,a^{\frac{1}{2}}}\right)^2 = \left(\sqrt[3]{a^{\frac{3}{2}}}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 = a$

34.  $\dfrac{3\sqrt{a}+2\sqrt{x}}{3\sqrt{a}-2\sqrt{x}} - \dfrac{3\sqrt{a}-2\sqrt{x}}{3\sqrt{a}+2\sqrt{x}} - \dfrac{24\sqrt{ax}}{9a-4x} =$

$= \dfrac{9a + 4x + 12\sqrt{ax} - 9a - 4x + 12\sqrt{ax} - 24\sqrt{ax}}{9a - 4x} = 0$

35.  $\dfrac{\sqrt[4]{1-x} + \dfrac{1}{\sqrt[4]{1+x}}}{1 + \dfrac{1}{\sqrt[4]{1-x^2}}} - \dfrac{\sqrt{1-x}}{\sqrt[4]{1-x}} = \dfrac{\dfrac{\sqrt[4]{1-x^2}+1}{\sqrt[4]{1+x}}}{\dfrac{\sqrt[4]{1-x^2}+1}{\sqrt[4]{1-x^2}}} - \dfrac{\sqrt{1-x}}{\sqrt[4]{1-x}} = \dfrac{\sqrt[4]{1-x^2}}{\sqrt[4]{1+x}} - \dfrac{(1-x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{4}}} =$

$= \dfrac{(1-x)^{\frac{1}{4}}(1+x)^{\frac{1}{4}}}{(1+x)^{\frac{1}{4}}} - (1-x)^{\frac{1}{4}} = \sqrt[4]{1-x} - \sqrt[4]{1-x} = 0$

36.  $\sqrt{\dfrac{1-a^3}{3-3a}} : \left(\sqrt{\dfrac{a+a^2+a^3}{1+a}} : \sqrt{3a^3}\right) = \dfrac{(1-a)^{\frac{1}{2}}(1+a+a^2)^{\frac{1}{2}}}{3^{\frac{1}{2}}(1-a)^{\frac{1}{2}}} \cdot \dfrac{(1+a)^{\frac{1}{2}}}{a^{\frac{1}{2}}(1+a+a^2)^{\frac{1}{2}}} \cdot 3^{\frac{1}{2}}\,a^{\frac{3}{2}} =$

$= (1+a)^{\frac{1}{2}} \cdot a = a\sqrt{1+a}$

37. $\sqrt{\dfrac{a^2 + ab}{xy}} \cdot \sqrt[4]{\dfrac{ab + b^2}{x}} : \sqrt{\dfrac{a + b}{xy}} : \sqrt[4]{\dfrac{a^3 b^2 + a^2 b^3}{x^2}} =$

$= \dfrac{a^{\frac{1}{2}} \, (a + b)^{\frac{1}{2}}}{x^{\frac{1}{2}} \, y^{\frac{1}{2}}} \cdot \dfrac{b^{\frac{1}{4}} \, (a + b)^{\frac{1}{4}}}{x^{\frac{1}{4}}} \cdot \dfrac{x^{\frac{1}{2}} \, y^{\frac{1}{2}}}{(a + b)^{\frac{1}{2}}} \cdot \dfrac{x^{\frac{2}{4}}}{a^{\frac{2}{4}} \, b^{\frac{2}{4}} \, (a + b)^{\frac{1}{4}}} = \dfrac{x^{\frac{1}{4}}}{b^{\frac{1}{4}}} = \sqrt[4]{\dfrac{x}{b}}$

38. $\sqrt[3]{x \sqrt{x}} \cdot \sqrt{x \sqrt{x \sqrt[3]{x}}} : \sqrt[3]{x} = \sqrt[3]{x^{\frac{3}{2}}} \cdot \sqrt{x \sqrt{x^{\frac{4}{3}}}} : x^{\frac{1}{3}} = x^{\frac{1}{2}} \cdot \sqrt{x^{\frac{5}{3}}} : x^{\frac{1}{3}} = x^{\frac{1}{2} + \frac{5}{6} - \frac{1}{3}} = x$