Calcola i seguenti limiti:

1. $$\lim_{x\to+\infty}\frac{1}{\sqrt{x+2}-\sqrt{x+5}}=\lim_{x\to+\infty}\frac{1}{\sqrt{x+2}-\sqrt{x+5}}\cdot\frac{\sqrt{x+2}+\sqrt{x+5}}{\sqrt{x+2}+\sqrt{x+5}}=\lim_{x\to+\infty}\frac{\sqrt{x+2}+\sqrt{x+5}}{-3}=-\infty$$

2. $$\lim_{x\to+\infty}(-x^4+3x^3-5x^2+x)=\lim_{x\to+\infty}(-x^4)=-\infty$$

3. $$\lim_{x\to-\frac{\pi}{2}}\left[sen\left(x+\frac{\pi}{2}\right)tgx\right]=\lim_{x\to-\frac{\pi}{2}}\left[\cos x\cdot\frac{senx}{\cos x}\right]=-1$$

4. $$\lim_{x\to-\infty}\frac{2x}{1-x}=\lim_{x\to-\infty}\frac{2x}{-x}=-2$$

5. $$\lim_{x\to-\infty}\frac{x+1}{|x|-1}=\lim_{x\to-\infty}\frac{x+1}{-x-1}=\lim_{x\to-\infty}\frac{x}{-x}=-1$$

6. $$\lim_{x\to+\infty}\frac{x^2+3x+1}{1-2x}=\lim_{x\to+\infty}\frac{x^2}{-2x}=-\infty$$

7. $$\lim_{x\to+\infty}\frac{x^2+3x+1}{x+2x^3}=\lim_{x\to+\infty}\frac{x^2}{2x^3}=0$$

8. $$\lim_{x\to-2}\frac{\sqrt[3]{x^3+8}}{\sqrt[3]{x^2-4}}=\lim_{x\to-2}\sqrt[3]{\frac{(x+2)(x^2-2x+4)}{(x-2)(x+2)}}=\lim_{x\to-2}\sqrt[3]{\frac{x^2-2x+4}{x-2}}=-\sqrt[3]{3}$$

9. $$\lim_{x\to-4}\frac{2x^2+7x-4}{3x^2+10x-8}=\lim_{x\to-4}\frac{(x+4)(2x-1)}{(x+4)(3x-2)}=\lim_{x\to-4}\frac{2x-1}{3x-2}=\frac{9}{14}$$

10. $$\lim_{x\to0}\frac{2-2\cos x}{x\,sen\,x}=2\lim_{x\to0}\frac{1-\cos x}{x\,sen\,x}\cdot\frac{1+\cos x}{1+\cos x}=2\lim_{x\to0}\frac{sen^2x}{x\,sen\,x\,(1+\cos x)}=2\lim_{x\to0}\frac{sen\,x}{x\,(1+\cos x)}=1$$

11. $$\lim_{x\to0}\frac{sen^2x-\cos x}{x^2}=\lim_{x\to0}\left(\frac{senx}{x}\right)^2-\lim_{x\to0}\frac{\cos x}{x^2}=-\infty$$

12. $$\lim_{x\to+\infty}[\log_2(x^2+2x)-\log_2(2x^2+3)]=\log_2\lim_{x\to+\infty}\frac{x^2+2x}{2x^2+3}=\log_2\lim_{x\to+\infty}\frac{x^2}{2x^2}=-1$$

13. $$\lim_{x\to0}\frac{sen\,x}{x^2+2x}=\lim_{x\to0}\frac{sen\,x}{x\,(x+2)}=\lim_{x\to0}\left(\frac{sen\,x}{x}\cdot\frac{1}{x+2}\right)=\frac{1}{2}$$

14. $$\lim_{x\to1^+}\frac{1}{1+2^{\frac{1}{x-1}}}=0\qquad\lim_{x\to1^-}\frac{1}{1+2^{\frac{1}{x-1}}}=1$$

15. $$\lim_{x\to1}\frac{x^3-2x+1}{x^3-1}=\lim_{x\to1}\frac{(x-1)(x^2+x-1)}{(x-1)(x^2+x+1)}=\frac{1}{3}$$

16. $\lim_{x\to 1^+} e^{\frac{x+2}{x-1}} = +\infty$

17. $\lim_{x\to +\infty} \left(1+\frac{4}{x}\right)^x = \lim_{x\to +\infty} \left[\left(1+\frac{1}{\frac{x}{4}}\right)^{\frac{x}{4}}\right]^4 = \boldsymbol{e^4}$

18. $\lim_{x\to +\infty} \left(\frac{x}{1+x}\right)^{-x} = \lim_{x\to +\infty} \left(\frac{1+x}{x}\right)^x = \lim_{x\to +\infty} \left(1+\frac{1}{x}\right)^x = \boldsymbol{e}$

19. $\lim_{x\to \frac{\pi}{2}} \left(1+\frac{1}{tg\,x}\right)^{tg\,x} = \boldsymbol{e}$

20. $\lim_{x\to 0} \frac{\ln(1-2x)}{x} = \lim_{x\to 0} \ln(1-2x)^{\frac{1}{x}} = \lim_{x\to 0} \ln\left[\left(1+\frac{1}{-\frac{1}{2x}}\right)^{\frac{1}{-2x}}\right]^{-2} = \ln e^{-2} = \mathbf{-2}$

Dopo aver determinato il dominio delle seguenti funzioni, individuane gli asintoti:

$$y=\frac{3x^2-1}{x-4} \qquad y=\frac{x+3}{x^2+4x+4} \qquad y=\ln\frac{x+1}{x-2}$$

$y=\frac{3x^2-1}{x-4}$ $\qquad D =]-\infty; 4\,[\ \cup\ ]\,4;\ +\infty[$

$\lim_{x\to\pm\infty}\frac{3x^2-1}{x-4} = \lim_{x\to\pm\infty}\frac{3x^2}{x} = \pm\infty$ $\qquad$ *può esistere asintoto obliquo*

$m = \lim_{x\to\pm\infty}\frac{\frac{3x^2-1}{x-4}}{x} = \lim_{x\to\pm\infty}\frac{3x^2-1}{x^2-4x} = \lim_{x\to\pm\infty}\frac{3x^2}{x^2} = 3$ $\qquad q = \lim_{x\to\pm\infty}\left(\frac{3x^2-1}{x-4}-3x\right) = \lim_{x\to\pm\infty}\frac{12x-1}{x-4} = 12$ $\qquad \boldsymbol{y = 3x + 12}$

$\lim_{x\to 4^\pm}\frac{3x^2-1}{x-4} = \pm\infty$ $\qquad$ *asintoto verticale* $\boldsymbol{x = 4}$

$y=\frac{x+3}{x^2+4x+4}$ $\qquad D =]-\infty; -2\,[\ \cup\ ]-2;\ +\infty[$

$\lim_{x\to\pm\infty}\frac{x+3}{x^2+4x+4} = \lim_{x\to\pm\infty}\frac{x}{x^2} = 0$ $\qquad$ *asintoto orizzontale* $\boldsymbol{y = 0}$

$\lim_{x\to -2^\pm}\frac{x+3}{(x+2)^2} = +\infty$ $\qquad$ *asintoto verticale* $\boldsymbol{x = -2}$

$y=\ln\frac{x+1}{x-2}$ $\qquad D =]-\infty; -1\,[\ \cup\ ]\,2;\ +\infty[$

$\lim_{x\to\pm\infty}\ln\frac{x+1}{x-2} = 0$ $\qquad$ *asintoto orizzontale* $\boldsymbol{y = 0}$

$\lim_{x\to -1^-}\ln\frac{x+1}{x-2} = -\infty$ $\qquad$ *asintoto verticale* $\boldsymbol{x = -1}$ $\qquad \lim_{x\to 2^+}\ln\frac{x+1}{x-2} = +\infty$ $\qquad$ *asintoto verticale* $\boldsymbol{x = 2}$