IL FATTORIALE

$$n! = n\,(n-1)(n-2)(n-3)\cdot \ldots \cdot 2\cdot 1 \qquad n \geq 2$$

$$0! = 1 \qquad 1! = 1$$

Proprietà:

$n! = n\,(n-1)!$

$(n+1)! - n! = n \cdot n!$

Dimostrazione: $(n+1)! - n! = (n+1)\,n! - n! = n!\,(n+1-1) = n \cdot n!$

COEFFICIENTI BINOMIALI

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{0} = 1 \qquad \binom{0}{0} = 1 \qquad \binom{n}{n} = 1$$

Legge delle classi complementari:

$$\binom{n}{k} = \binom{n}{n-k}$$

Formula di ricorrenza

$$\binom{n}{k+1} = \binom{n}{k} \cdot \frac{n-k}{k+1}$$

Dimostrazione:

$$\binom{n}{k} \cdot \frac{n-k}{k+1} = \frac{n!}{k!\,(n-k)!} \cdot \frac{n-k}{k+1} = \frac{n!}{(k+1)\,k! \cdot (n-k) \cdot (n-k-1)!}(n-k) = \frac{n!}{(k+1)!\,(n-k-1)!} = \binom{n}{k+1}$$

Formula di Stifel

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Dimostrazione:

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)!\,(n-k)!} + \frac{(n-1)!}{k!\,(n-k-1)!} = (n-1)!\,\frac{k+n-k}{k!\,(n-k)!} = \frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$

BINOMIO DI NEWTON

$$(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^{n-k} B^k$$