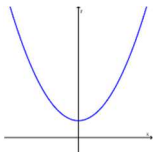
$ax^2 + bx + c = 0$ EQUAZIONE DI SECONDO GRADO RIDOTTA ALLA FORMA NORMALE

Le soluzioni di un'equazione di secondo grado $ax^2 + bx + c = 0$ costituiscono
le ascisse dei **punti di intersezione** tra la parabola di equazione $y = ax^2 + bx + c$ e l'asse x

EQUAZIONI INCOMPLETE[1]:

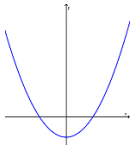
1. $b = 0 \quad \wedge \quad c \neq 0$: **EQUAZIONE PURA** $\qquad ax^2 + c = 0$

$a \cdot c > 0 \quad \Rightarrow \quad x \in \mathbb{C}$



La parabola non interseca l'asse x.
Il vertice della parabola di trova sull'asse y.

$a \cdot c < 0 \quad \Rightarrow \quad x = \pm\sqrt{-\frac{c}{a}}$

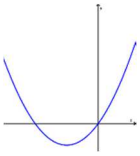


La parabola interseca l'asse x.
Il vertice della parabola di trova sull'asse y.

2. $b \neq 0 \quad \wedge \quad c = 0$: **EQUAZIONE SPURIA** $\qquad ax^2 + bx = 0$

Ammette sempre due soluzioni reali e distinte e una è nulla

$$x_1 = 0 \qquad x_2 = -\frac{b}{a}$$

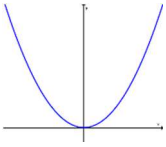


La parabola passa per l'origine degli assi

3. $b = c = 0$: **EQUAZIONE MONOMIA** $\qquad ax^2 = 0$

Ammette sempre due soluzioni reali e coincidenti entrambe nulle

$$x_{1,2} = 0$$



La parabola ha il vertice nell'origine degli assi

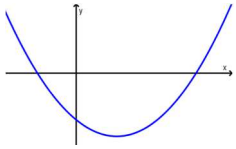
EQUAZIONI COMPLETE:

Calcolo $\qquad \Delta = b^2 - 4ac$

1. $\Delta > 0$

Due soluzioni reali e distinte:

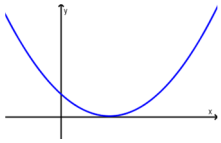
$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$



2. $\Delta = 0$

Due soluzioni reali coincidenti:

$$x_{1,2} = -\frac{b}{2a}$$

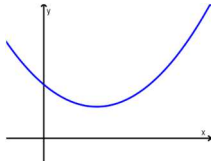


La parabola ha il vertice sull'asse x

3. $\Delta < 0$

Due soluzioni complesse e coniugate:

$$x_{1,2} = \frac{-b \pm i\sqrt{\Delta}}{2a}$$

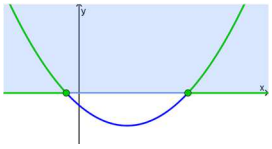
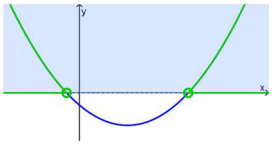
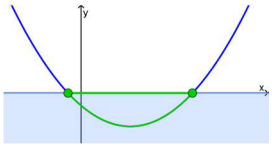
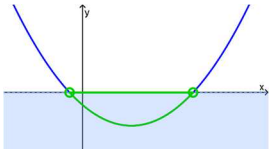
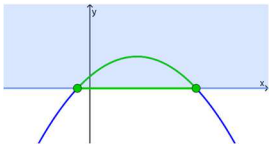
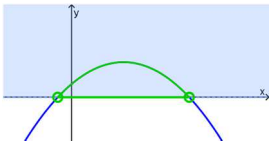
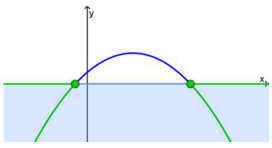
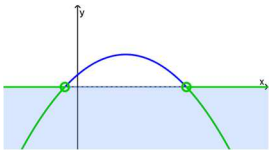


La parabola non interseca l'asse x

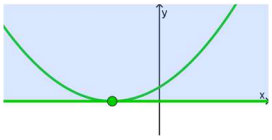
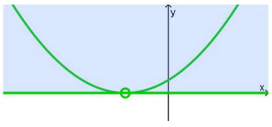
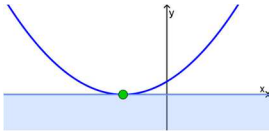
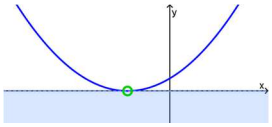
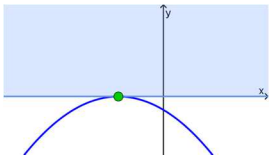
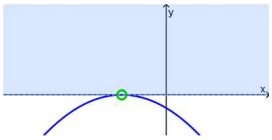
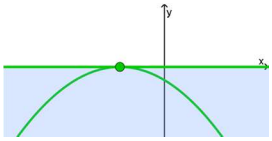
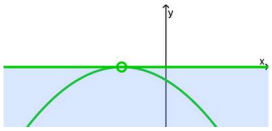
[1] Per comodità verrà sempre considerato il caso $a > 0$; la situazione è simile nel caso $a < 0$, solo che le parabole hanno tutte la concavità rivolta verso il basso.

DISEQUAZIONI COMPLETE[2]:

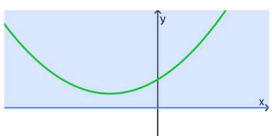
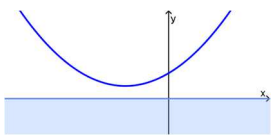
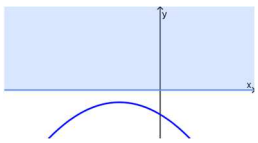
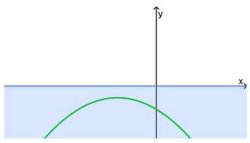
$\Delta > 0$: $x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ $\qquad x_1 < x_2$

$a > 0$	$ax^2+bx+c \geq 0$  $x \leq x_1 \ \vee \ x \geq x_2$	$ax^2+bx+c > 0$  $x < x_1 \ \vee \ x > x_2$	$ax^2+bx+c \leq 0$  $x_1 \leq x \leq x_2$	$ax^2+bx+c < 0$  $x_1 < x < x_2$
$a < 0$	$ax^2+bx+c \geq 0$  $x_1 \leq x \leq x_2$	$ax^2+bx+c > 0$  $x_1 < x < x_2$	$ax^2+bx+c \leq 0$  $x \leq x_1 \ \vee \ x \geq x_2$	$ax^2+bx+c < 0$  $x < x_1 \ \vee \ x > x_2$

$\Delta = 0$: $x_{1,2} = -\dfrac{b}{2a}$

$a > 0$	$ax^2+bx+c \geq 0$  $\forall x \in \mathbb{R}$	$ax^2+bx+c > 0$  $\forall x \in \mathbb{R} - \{x_1\}$	$ax^2+bx+c \leq 0$  $x = x_1$	$ax^2+bx+c < 0$  $\nexists x \in \mathbb{R}$
$a < 0$	$ax^2+bx+c \geq 0$  $x = x_1$	$ax^2+bx+c > 0$  $\nexists x \in \mathbb{R}$	$ax^2+bx+c \leq 0$  $\forall x \in \mathbb{R}$	$ax^2+bx+c < 0$  $\forall x \in \mathbb{R} - \{x_1\}$

$\Delta < 0$

$a > 0$		$a < 0$	
$ax^2+bx+c \geq 0$ $ax^2+bx+c > 0$  $\forall x \in \mathbb{R}$	$ax^2+bx+c \leq 0$ $ax^2+bx+c < 0$  $\nexists x \in \mathbb{R}$	$ax^2+bx+c \geq 0$ $ax^2+bx+c > 0$  $\nexists x \in \mathbb{R}$	$ax^2+bx+c \leq 0$ $ax^2+bx+c < 0$  $\forall x \in \mathbb{R}$

[2] Il caso delle disequazioni incomplete è compreso in quello delle equazioni complete con $\Delta > 0$ per le pure (con soluzioni reali) e le spurie, in quello delle equazioni complete con $\Delta = 0$ per le monomie e in quello delle equazioni complete con $\Delta < 0$ per le pure senza soluzioni reali